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Pricing Critical Illness Insurance Premiums Using Multiple State Continous Markov Chain Model

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Pricing Critical Illness Insurance Premiums Using Multiple State Continous Markov Chain Model

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Abstract. Critical Illness Insurance is an insurance product with a lump sum benefit or cash payment if the policyholder is diagnosed with critical illness in an insurance contract. The health state change process can be observed and modeled by a multi-state Markov Chain with a time-continuous parameter. In this article, we will illustrate how the mathematics of Markov Chain can be used to develop a model of state change in critical illness in the case of a cancer patient. Health state changes process in critical illness modeled by four states Markov chain. Using transition probability which is based on transition intensity of continuous Markov chain. We will estimate the transition intensity and used it in a differential equation of transition probability. The application of Markov chain model will be used to estimate the value of the premiums in some of critical illness insurance benefit model. The premiums value will be shown in the table in section five.

Keywords: Multiple state model; Continous Markov chain; Cancer; Transition intensity; Transition probability; Critical illness insurance premium.

1. Introduction

Health insurance in developing countries currently in a declining condition. Baione and Levantesi (2014) in their study explain the cause of obstacle in the health insurance market in Italy is the limited availability of data and the unsustainable survey by the National Institute of Statistics abbreviated as ISTAT. The same thing happened in Indonesia as one of the developing countries. Data on national health statistics are available is limited and difficult to obtain. Also, the form of aggregate data from mortality, morbidity or prevalence is available. Meanwhile, to build an accurate model of health insurance, comprehensive statistical data is needed. Statistical data of sickness is very useful in modeling a health insurance product. The data needed in the form of the number of people with a disease in a population, mortality rate data caused by a disease, as well as grouping regular and consistent disease data. But the fact in developing countries where the rate of occurrence of the disease is quite high, the statistical data held is limited.

Health insurance modeling is done by using a probabilistic approach using a multi-state model. Several health insurance companies can use this model including disability insurance, long-term disease insurance (*Long Term Care)* and critical illness *(critical illness)* or known as CI. Critical illness insurance is a health insurance product that offers protection to policyholders if the policyholders have a chronic disease. The benefits provided are in the form of cash coverage that can

be used by policyholders for medical expenses for chronic illnesses suffered. This will certainly provide financial insurance for this insurance policyholder. This insurance begins to develop along with the increasing incidence of chronic diseases in the community.

Chronic disease is a disease that needs special attention and requires long-term health care and is difficult to return to normal (Warshaw, 2006). According to the American National Statistics Center, chronic diseases usually last for three months or more by the National Center for Health Statistics. As a result, chronic diseases absorb large costs. Commonly identified chronic diseases are coronary heart disease, cancer, stroke, and kidney failure classified as non-communicable diseases. According to data from the World Health Organization (2018), the global cancer burden is estimated to rise to 18.1 million new cases and leads to 9.6 million deaths in 2018 makes canser as the second leading cause of death globally. Increasing cancer is related to population growth and several causes of cancer linked to social-economic conditions. It shows that cancer-related to poverty and associated with lifestyles more typical of industrialized countries.

Based on these limitations on statistical data of sickness, Baione and Levantesi (2014) provide an alternative in modeling medical insurance with limited data using a multi-state model applied to Critical Illness insurance. The multi-state method already applied to many health insurance cases. Multi-state are widely used in actuarial science since they provide a convenient way of representing changes in people's health statuses and made calculation easy if we assume that the model follows Markov Chain (Zhang, 2016). On that paper, Zhang provides a semi-Markov model to captured heterogeneity as the result of unobservable factors that affect an individual's health. Multi-state models are one of the most exciting developments in actuarial science nowadays. In their paper, Gogola and Kopecka (2017) describe the actuarial structure of disability insurance, long-term insurance, and critical illness insurance cover. Actuarial problems such as pricing and reserving are considered within the context of multi-state modeling, providing a strong and sound framework.

A multi-state model assuming that there is a pattern random on change circumstances of an individual's health status. The randomization that occurred depends on time and the number of situations of an individual's life cycle in period contract insurance. A process carried out when we observe individuals or groups involving time called stochastic process. A stochastic process is a collection of random variable $\{Z(t), t \in T\}$. This sequence of a variable has parameter index time t. The main purpose of modeling critical illness insurance is to determine the correct stochastic model for a random pattern of the individual's situation. The stochastic model that is considered most appropriate for these multi states is the Markov Chain. Previous research shows almost all health insurance modeling with multi-state using the Markov chain method. Besides, modelling using the Markov Chain produce models that are accurate and easily adapted (Christiansen, 2012).

2. Markov Chain

Markov Chain is defined as a stochastic process with continuous index parameter $\{Z(t), t \in T\}$ with a set of times $t_1 < t_2 < \cdots < t_n$ so the probability of $Z(t_n)$ given $Z(t_1), Z(t_2), \ldots, Z(t_n)$ depends on $Z(t_{n-1})$ (Taylor dan Karlin, 1998). Based on the above definition, in the Markov Chain, the event occurs in the past does not affect future events, so future events only affected by current events. Characteristics of the Markov chain define the conditional opportunity of the current event that can be used to estimate future events based on current events known. Examples of the Markov Chain application are observing an individual's health status by looking at the displacement at a certain time interval. If $Z(t_n) = i_n$ we can write the process as:

$$P(Z(t_n) = i_n | Z(t_1) = i_1, Z(t_2) = i_2, \ldots, Z(t_{n-1}) = i_{n-1})$$

$$= P(Z(t_n) = i_n | Z(t_{n-1}) = i_{n-1}) \tag{2.1}$$

In the Markov Chain, there is a finite state space $Z = \{1, 2, \ldots\}$, which is called a discrete state space and contains a set of possible values for Z. Another characteristic of the Markov Chain can be

distinguished based on the time parameter index T discrete-time and continuous. Markov Chain with $T = \{0,1,\dots\}$ is called Markov Chain with discrete-time, whereas if $T = [0, \infty)$ is called Markov Chain with continuous time.

In modeling a phenomenon as a continuous-time Markov model, it is assumed that the process involved follows Markov's properties as the conditional probability of such an event, that is the future change depends on the current event (Koller, 2010). For $u, t \geq 0$ and $i, j, z(s), 0 \leq s \leq u$ can be written mathematically as:

$$P(Z(t + u) = j | Z(u) = i, Z(s) = z(s))$$

$$= P(Z(t + u) = j | Z(u) = i) \tag{2.2}$$

In a continuous-time Markov chain, it is necessary to note the close relationship with the Poisson process. The Poisson process has a rate or intensity rate that appears in continuous-time Markov Chain. Poisson process or counting process $\{N(t), t \geq 0\}$ is a simple stochastic process that explains several events occurs at intervals time t. According to Taylor and Karlin (1975), a stochastic process can be said to be a Poisson process with λ parameter and $\lambda \geq 0$ if:

(i) $N(0) = 0$

(ii) The process has independent and stationary increments

(iii) $P\{N(t) = 1\} = \lambda t + o(t)$

(iv) $P\{N(t) \geq 2\} = o(t)$

where f is a function $o(t)$ if $\lim\limits_{t \to 0} \frac{f(t)}{t} = 0$.

The close relationship between continuous-time Markov Chains and Poisson Processes is explained by the similarity of distribution of waiting times between events. In the Markov Chain of continuous-time, the length of an individual in state i before transitioning to state j is a random variable. This random variable has an exponential distribution with parameter λ.

3. Transition Probability and Transition Intensities of Continous Markov Chain in Actuarial Context

In sections 1 and 2 we already talk about the use of a multi-state model in Markov Chain. Multi-state models provide a solid foundation for pricing and valuing complex insurance contracts. Each state in Markov Chain corresponds to an event that determines the cash flows (premiums and benefits). In equation (2.2) suppose that $= x$, notation x is the realization of the stochastic process $\{Z(t)\}$. However, in the actuarial field and later in this study, x notation will be used to express the age of a person. In the actuarial field, the probability for individual age x expected to live for t years is denoted as $_t p_x$. Combining all definitions above with the transition probability in continuous Markov Chain, an additional index i and j are used in the notation $_t p_x$. Thus, the transition probability for continuous Markov Chain with discrete state space is defined as:

$$P\big(Z(t + x) = j \mid Z(x) = i\big) = {}_t p_x^{ij}; i, j = 1, 2, \cdots n \tag{3.1}$$

The simplest model of Markov Chain represents a single life theory where an individual is in two possible states; alive and dead.
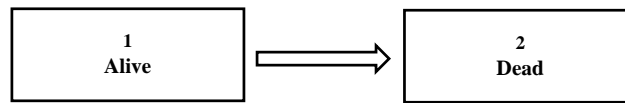
Figure 1. Set of States of Transitions for Simple Multi-State Model

The premium payment is made if an individual remains in state 1 (alive) and the benefit is made if an individual made the transition from state 1 to state 2 (death). This simple model captures all the life contingent information for the calculation of actuarial values that only depend on whether a single individual is alive oran dead at a given age. Based on the simple alive-dead model, additional states are considered to analyze more complicated models. For critical illness cases, the alive state in Figure 2 can be split into two transient states which, in an application, correspond to occurrences of various medical status as critical illness conditions. The same goes for the death states that can be split into two absorbent states such as death due to critical illness and death due to other causes.
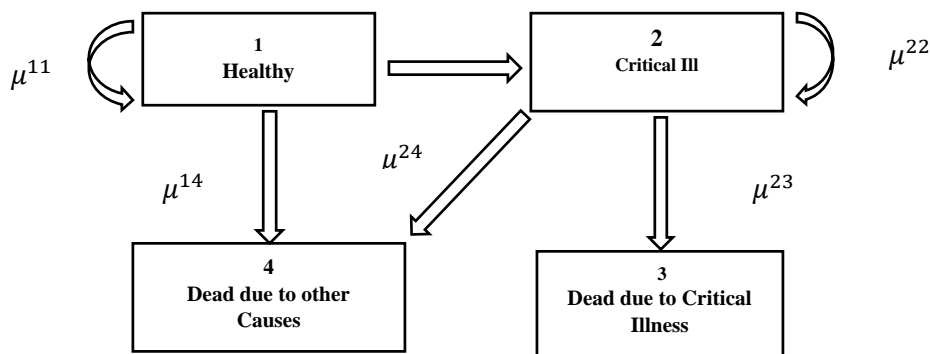

Figure 2. Set of States of Transitions for Critical Illness Case

Based on Markov Chain state-space above, the transition probability notation used in this study:

${}_t p_x^{11}$: the probability of individual age x currently in health state, at age $x + t$ still in health state.

${}_t p_x^{12}$: the probability of individual age x currently in health state, at age $x + t$ move to the critically ill state.

${}_t p_x^{14}$: the probability of individual age x currently in health state, at age $x + t$ move to die due to other causes state.

${}_t p_x^{22}$: the probability of individual age x currently in critical illness state, at age $x + t$ still in critical illness stage.

${}_t p_x^{23}$: the probability of individual age x currently in critical illness state, at age $x + t$ move to die because of the critical illness state.

${}_t p_x^{24}$: the probability of individual age x currently in critical illness state, at age $x + t$ move to die due to other causes state.

By the transition probability above we also have a pair of transition intensities. Transition intensity for each pair of states i and j with $i \neq j$ is the multiplication of the rate at the time of the process leaving the state i with the transition probability to state j (Ross, 2000). Also, transition intensity (rates) represent speeds of the changes (Zhang, 2016). The mathematical expression can be written as:

$$\mu_x^{ij} = \lambda_i \cdot {}_t p_x^{ij}, i \neq j \tag{3.2}$$

For each pair of states i and j with $i \neq j$, the transition intensities from state i to state j are

$$\mu_x^{ij} = \lim_{t \to 0^+} \frac{{}_t p_x^{ij}}{t} \tag{3.3}$$

The equation (3.2) and (3.3) appears in Haberman and Pittaco (1999), Gerber (1997) and Bowers (1997). The equation above can be proved by the events of two or more transitions in time interval $[x, x+t]$ by $o(t)$. So that it will be equivalent to:

$$_t p_x^{ij} = t \cdot \mu_x^{ij} + o(t) \; ; i \neq j \tag{3.4}$$

where $\lim_{t \to 0^+} \frac{o(t)}{t} = 0$.

There are some properties in the Markov chain which are irreducible if there is the same class and each state communicates with each other. If two conditions communicate with each other then it is said i↔j. Whereas if the situation does not communicate then $p_x^{ij} = 0$ for $x \geq 0$ or $p_x^{ji} = 0$ for $x \geq 0$. The condition i is called absorption if $_t p_x^{ij} = 0, i \neq j \forall t$ and consequently $_t p_x^{ii} = 1, \forall t$. Based on these properties we know that **Figure 3** has two absorption states (State 3 and State 4) and the other is a pure transient state (State 1 and State 2).

4. The Chapman-Kolmogorov Equation

Based on Markov's property, the corresponding transition probabilities in Markov Chain are known as solutions of the Chapman-Kolmogorov equation. In a continuous-time Markov chain with a discrete state space, the Chapman-Kolmogorov equation is defined as follows:

$$_{t+h} p_x^{ij} = \sum_{k=1}^{n} p^{ik}(x, x+t) p^{kj}(x+t, x+t+h) \; ; i, j \in \{1, 2, \cdots, n\} \tag{4.1}$$

The differential equations for transition probabilities according to the model in Figure 2 and based on the Chapman-Kolmogorov equation above are:

$$\frac{d}{dt} {}_t p_x^{11} = -{}_t p_x^{11} \left(\mu_{x+t}^{12} + \mu_{x+t}^{14} \right) \tag{4.2}$$

$$\frac{d}{dt} {}_t p_x^{12} = {}_t p_x^{11} \cdot \mu_{x+t}^{12} - {}_t p_x^{12} \left(\mu_{x+t}^{23} + \mu_{x+t}^{24} \right) \tag{4.3}$$

$$\frac{d}{dt} {}_t p_x^{14} = {}_t p_x^{11} \cdot \mu_{x+t}^{14} + {}_t p_x^{12} \cdot \mu_{x+t}^{24} \tag{4.4}$$

$$\frac{d}{dt}\,{}_t p_x^{22} = -{}_t p_x^{22}\left(\mu_{x+t}^{23} + \mu_{x+t}^{24}\right) \tag{4.5}$$

$$\frac{d}{dt}\,{}_t p_x^{23} = {}_t p_x^{22} \cdot \mu_{x+t}^{23} \tag{4.6}$$

$$\frac{d}{dt}\,{}_t p_x^{24} = {}_t p_x^{22} \cdot \mu_{x+t}^{24} \tag{4.7}$$

5. Critical Illness Premium

Premium is a payment or a series of payments by the policyholders so that an insurance policy contract continues to protect the policyholders. Premium payments are calculated based on the present value. In this study net single premium will be used so that premium payments are made once and the additional costs for policy-making are not included in the calculation. In general, the calculation of an insurance premium is based on three things, death probability, interest rates, and costs. However, because what is used is the net single premium so the amount of additional costs for policymaking is not included.

Critical Illness insurance usually takes the form of a rider offered a combination with a life policy. A basic distinction has to be made between the two types of CI Insurance. The first and most common form of cover provides only a prepayment on the sum insured of an underlying policy. The second provides for additional benefits without affecting the life sum insured of the main policy (additional benefit) (Gyimah, 2011).

Formulas to calculate pure premium rates:

(i) Stand Alone Policy with N duration limit. In the event of CI claim, an additional benefit is due to the underlying main insurance continuing unaltered.

$$\overline{A}_{x:n}^{(SA)} = \int_0^n {}_t p_x^{12} \cdot \mu_x^{12}(t) \cdot v^t\,dt \tag{5.1}$$

(ii) Critical Illness Acceleration Benefit provides a combination of a death benefit and a critical illness cover. Payment is made either when the policyholder dies or diagnosed as having one of the conditions specified in the policy. For example, when CI benefit is attached to whole life insurance or endowment, a CI claim would simply be bringing forward the payment that would ultimately have been made on death.

$$\overline{A}_{x:n}^{(AB)} = \int_0^n {}_t p_x^{11} \cdot \mu_x^{12}(t) \cdot v^t + \int_0^n {}_t p_x^{11} \cdot \mu_x^{14}(t) \cdot v^t\,dt \tag{5.2}$$

Based on equation (4.2) it can be seen that the first part of the integral is the equation of CI with the Stand Alone model. So that the Critical Illness insurance model with Acceleration Benefit is Stand Alone insurance with the calculation of additional life insurance.

6. Numerical Result

We use transition intensities calculation from CMI working paper 50 and following Gutierrez and MacDonald (2003) rates for cancer were found for:

$$\mu_x^c = \exp(-11.25 + 0.105x) \text{ for } (x < 51)$$

with linear interpolation between ages 51-60,

$$\mu_x^c = \exp(-10.78 + 0.123x - 0.00033x^2) \text{for } (x < 53)$$

$$\mu_x^c = -0.01545632 + 0.0003805097\, x \text{ for } (x \geq 53)$$

$$\mu_x^c = \exp(0.2591585 - 0.01247354\, x + 0.0001916916\, x^2 - 8.952933 \cdot 10^{-7}\, x^3)\,(x \geq 60)$$

Table 5.1 Estimation Value of Transition Intensity of Critical Patient by Age Group

Age	$\hat{\mu}^{12}$	$\hat{\mu}^{23}$	$\hat{\mu}^{14}$	$\hat{\mu}^{24}$
(15,24)	0.107	0.452	0.103	0.402
(25,34)	0.106	0.427	0.101	0.427
(35,44)	0.105	1.097	0.094	0.774
(45,54)	0.109	2.132	0.094	1.865
(55,64)	0.107	0.963	0.091	0.668

Based on Table 5.1 we could see that patient with age group 15 to 24 has transition intensity from healthy to critical illness with value 0.107, transition intensity from critical ill state to die because of the ill is 0.452. To find the transition probability estimation, we use the Chapman-Kolmogorov equation in section 3, by the substitute that equation we got this mathematical expression:

$$_t p_x^{11} = \exp\left(-\int_0^t \mu_x^{12}(u) + \mu_x^{14}(u)du\right)$$

If we want to find transition probability of age (15,24) with $t = 1$

$$_t p_x^{11} = e^{-(\mu^{12}+\mu^{14})}$$

$$= e^{-(0.107+0.103)} = 0.809$$

After obtaining transition probability estimation and transition intensity estimation, we can calculate the amount of premium of Critical Illness Insurance of each age group. If we have a lump sum benefit for Rp.100.000.000, we show the premium in a 1-year contract of Stand Alone policy model of insurance with a 6% rate of interest p.a in Table 5.2 below:

Table 5.2 Stand Alone Policy Premium of Critical Illness Insurance by Age Group

Age	Premiums
(15-24)	Rp. 9,006,200
(25-34)	Rp. 9,297,000
(35-44)	Rp. 9,260,000
(45-54)	Rp. 9,630,000
(55-64)	Rp. 9,434,000

7. Conclusions and Discussions

In this study, we propose a multiple state model of continuous Markov Chain to determine the premium of Critical Illness insurance. It is very crucial to know about the number of states you will use to calculate the premium. We have presented an application of multi-state models to problems in the actuarial field in health insurance. There are various extensions that we could apply in multi-state models. The amount of premium obtained is very related to the number of state space in multiple states and also the transition intensities. We can add the cancer stage conditions as the extensions of the critical illness state, but the data about an individual's cancer stage is very limited. Also to allow transition state not only depends on an individual's current age but also how long they have been in the current state. It will lead to a semi-Markov process. Zhang (2016) already proposed this method to captured heterogeneity as the result of unobservable factors that affect an individual's health.

As for a numerical solution, we use rate transition based on CMI working paper 50 and following Gutierrez and MacDonald (2003) and still there is another approach for calculating rate transition such as using Gompertz-Makeham law method. Note that there is no industry standard model for CI insurance so there is a need for awareness of model risk when assessing an insurance policy especially for the long term.

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