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Number of Failures for Weibull Hazard Function witha Fuzzy Shape Parameter

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Abstract. Number of failures play important role in maintenance strategy for industrial equipment, either repairable or non-repairable ones. There are several form of known distribution used to model the failures of an industrial equipment. Weibull distribution, and its hazard function, is among the most used distribution. Most maintenance models in literature mainly consider certain or crisp condition in a deterministic form. However, many real phenomena seem do not suitable to model in such certain or crisp condition. One approach to model a possibilistic uncertainty is by applying the fuzzy number theory. In this paper, we discuss the Weibull hazard function by assuming a fuzzy shape parameter. We look for the number of failures generating by the function using two different methods. The first one assumes that the fuzziness of the shape parameter propagates to the number of failures with the same form of fuzzy number membership. The second one use the fuzziness of the shape parameter in the computation of the number of failures directly, through the concept of alpha-cut or alpha-level. Some comparisons regarding these approaches are presented.

1. Introduction

Number of failures play important role in maintenance strategy for industrial equipment, either repairable or non-repairable ones [1,2]. There are several form of known distribution used to model the failures of an industrial equipment. Weibull distribution, and its hazard function, is among the most used distribution, either the two parameters model or the three parameters model [3,4]. Most maintenance models in literature mainly consider certain or crisp condition in terms of uncertainty [5-7]. However, many real phenomena seem do not suitable to model in such certain or crisp condition. The importance of possibilistic uncertainty is briefly reviewed [8]. One approach to model a possibilistic uncertainty is by applying the fuzzy number theory [9,10]. Fuzzy theory is now getting popular to use in reliability maintenance literatures [11]. Some new models of this category of uncertainty are begin to appear, such as [12].

In this paper, we discuss the Weibull hazard function by assuming a fuzzy shape parameter. A fuzzy shape parameter is modelled by Crisp function as one of fuzzy equation. As a fuzzy equation we can look Crisp function as, (i) function with fuzzy constraint, (ii) function that propagates the fuzziness of independent variable to dependent variable, and (iii) fuzzy function.

Here we look for the number of failures generating by the function using two different methods [13]. The first one assumes that the fuzziness of the shape parameter propagates to the number of failures with the same form of fuzzy number membership. Examples of works in this category are [14]. The

second one uses the fuzziness of the shape parameter in the computation of the number of failures directly, through the concept of alpha-cut or alpha-level [15]. Some comparisons regarding the results of these approaches are also presented based on the latest methodology given in [16].

2. Methods

2.1. Model Derivation Crisp and fuzzi weibull hazard function

In this section we present a brief introduction to fuzzy numbers. In general, a fuzzy number is a generalization of a regular, real number in the sense that unlike in the case of a regular number, where it refers to one single value (crisp), a fuzzy number refers to a connected set of possible values (*A*), where each possible value of A, say *a*, has its own weight in the interval [0,1]. This weight is called the membership function, usually written as $\mu : a \in A \rightarrow x \in [0,1]$. This fuzzy number sometimes written with the symbol $\tilde{A} = (A, \mu(A))$ or alternatively $\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) \middle| x \in X \right\}$ representing the underlying connected set with the membership function $\mu(A)$. In this regard, the fuzzy number is viewed as a coupled comprising of a set together with its grade or membership function. The fuzzy number is designed to represent the uncertainty, unclear and inaccuracies of the abundance of information available.

There are several functional approaches that can be used to define membership values. The membership functions can be classified into two groups, the one formed by a linear functional and the other one formed by non-linear functional. Examples of the most elementary form of a non-trivial fuzzy number are the Triangular Fuzzy Number (TFN) which is often written as (a;b;c) and the Trapezoidal Fuzzy Number (TrFN) which is often written as (a;b;c;d). The functional forms are given as follow, which graphically shown in Figures (1) and (2).

- The triangular curve representation (TFN):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \leq a \\ \dfrac{x-a}{b-a} & ; a \leq x \leq b \\ \dfrac{x-c}{b-c} & ; b \leq x \leq c \\ 0 & ; x \geq c \end{cases}. \tag{1}$$

- The trapezoidal curve representation (TrFN):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \leq a \\ \dfrac{x-a}{b-a} & ; a \leq x \leq b \\ 1 & ; b \leq x \leq c \\ \dfrac{x-d}{c-d} & ; c \leq x \leq d \\ 0 & ; x \geq d \end{cases}. \tag{2}$$

In TFN, *b* is called the *core* and the sets [*a,b*) and (*b,c*] are called the *support* of the fuzzy number, while In TrFN the core is given by [*b,c*] and the support is given by the set [*a,b*) and (*c,d*]. Other forms of fuzzy numbers are piecewise quadratic fuzzy number [17], pentagonal [18], Bell shaped [19], parabolic trapezoidal [20], new bell shaped [21], and many others. For simplicity all examples in this paper will assume the triangular fuzzy numbers. In the next section we briefly describe the *alpha-cut* or *α-cut* of a fuzzy number.
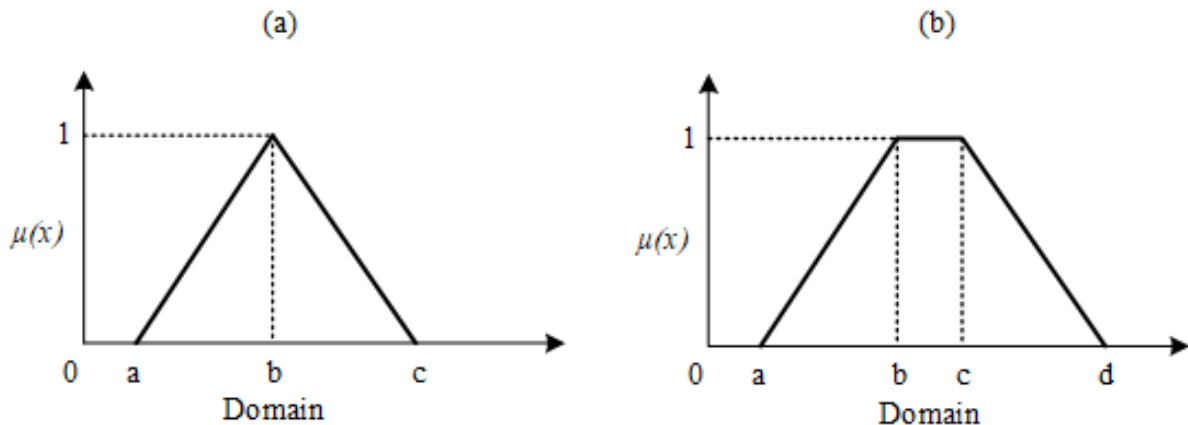
Figure 1. Graphical representation of a triangular fuzzy number ($a;b;c$) - left, and a trapezoidal fuzzy number ($a;b;c;d$) –right.

2.2. The α-cut of a fuzzy number

The *α-cut* of a fuzzy number or fuzzy set sometimes is called the *α-level set*. It is defined as the objects in the associated fuzzy set which have the membership function more than or equal *α*. This is a crisp set. The set is called a *strong α-level set* if the membership of the objects more than *α*. As an example, if A={(a_1,(0.1) , (a_2,0.2) , (a_3,0.3),(a_4,0.4), (a_5,0.3) , (a_6,0.2) , (a_7,0.1)}, then 0.2-level set of A is { a_2, a_3, a_4, a_5, a_6}. It is easy to show that the *α-cut* of the triangular fuzzy number (1) is given by:

$$A_\alpha = \left[a_1^\alpha, a_2^\alpha \right] = \left[(b-a)\alpha + a, (b-c)\alpha + c \right] \tag{3}$$

for all $\alpha \in [0,1]$.

2.3. Generalized mean value defuzzification

Defuzzification is the process of converting a fuzzified output into a single crisp value with respect to a fuzzy set.

In this section will be discuss There are some known methods of defuzzification, such as fuzzy mean, center of gravity, last of maximum, last of maxima etc [22]. In this paper we introduce the generalized mean value defuzzification method as the following. Let a TFN is given by $\tilde{A} = (a;b;c)$, the generalized mean value defuzzification (GMVD) is defined by

$$N(\tilde{A}) = \frac{a + nb + c}{n + 2} \tag{4}$$

The GMVD has the following properties as described in the Theorem. We will use this method for comparing the fuzzy output in this paper.

Theorem 1:

1. For a symmetrical case, i.e. $b - a = c - b = p$ then $N(\tilde{A}) = b$
2. For an asymmetrical case, i.e. $b - a = p \neq c - b = q$ then
   a. $N(\tilde{A}) > b$ if $p < q$
   b. $N(\tilde{A}) < b$ if $p > q$
3. If $n \to \infty$ then $N(\tilde{A}) = b$ regarless the value of p and q

The proof is clear.

2.4. Number of failures for weibull hazard function with fuzzy parameter

We consider the following Weibull Cumulative Distribution, g, and Weibull Hazard Function, h:

$$g(t) = 1 - e^{-t^{\beta}} , \tag{5}$$

and

$$h(t) = \beta t^{\beta-1} . \tag{6}$$

Hazard function play important roles in many areas, such as epidemiology [23,24] and reliability engineering [25]. In reliability the hazard function is associated with the number of failures for an equipment under investigation, or in a slightly form, it is the lifetime of the equipment [25].

The parameter β is the shape parameter of the function. In this paper we consider this parameter is a fuzzy number. We will investigate the number of failures for a given fuzzy shape parameter in the Weilbull distribution above. The distribution function is crisp in nature, but we will consider the fuzziness of the shape parameter and its effect to the number of failures following the distribution. We will treat the fuzziness of the shape parameter in two different approaches.

In the first approach we will use the TFN as a representation of the fuzzy shape parameter β. The TFN is identified by three numbers a, b, and c satisfying equation (1). We compute the number of failures by plugging these three crisp parameters to obtain three crisp output, say a', b', and c'. By assuming the same fuzzy measure propagates to the output, we have μ(a')= μ(a), μ(b')= μ(b), and μ(c')= μ(c), which constitute a TFN fuzzy output (a';b';c').

In the second approach, the fuzzy number A_α in equation (3) is approximated by a sequence of interval associated with the number α in [0,1]. This sequence is a crisp number in the interval indicating the support of the fuzzy number for every α in [0,1). If α is 1 then the support collapses into the core of the fuzzy number. The computation to obtain the number of failures is done at the end points of the interval. Hence the stack of the end points of the intervals need not to be a triangular fuzzy number, however in many cases it forms a TFN-like (see numerical examples for the details).

3. Results and Discussion

To illustrate the concept above we use two different values for the shape parameters, the relatively small value $\tilde{\beta} = (p = 1.25; q = 1.55; s = 1.85)$ and the relatively large value $\tilde{\beta} = (p = 2.50; q = 2.75; s := 2.80)$. The graphs of these TFNs are shown in Figure 1. The number of failures for the shape parameters in Figure 1 at *t*=10 is presented in Figure 2. In general, we can conclude that using this method the fuzziness of the shape parameter has made the fuzziness of the number of failures increases (in terms of the length of its base). Further conclusion that can be derived is that both methods give the same base for the fuzzy number of failures but with different degree of membership.
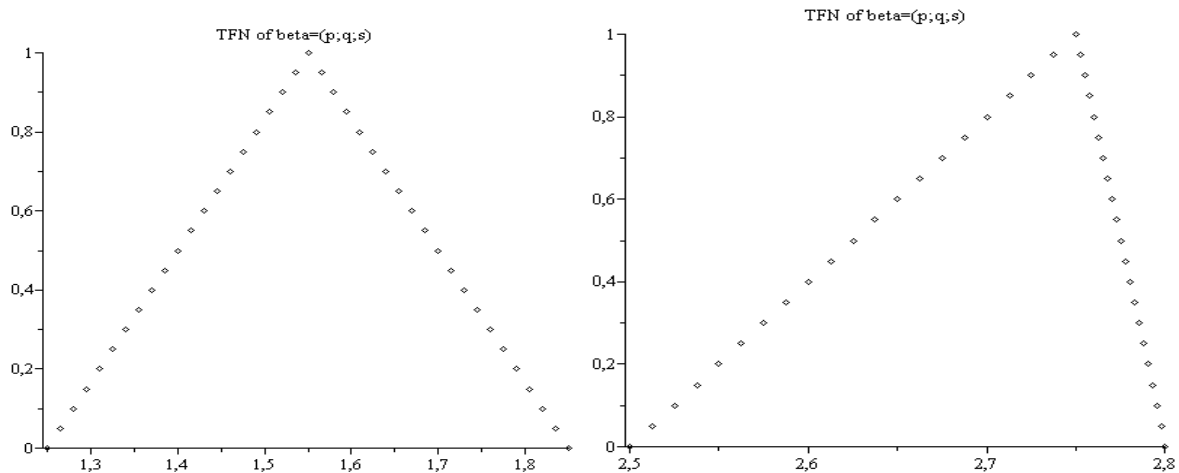
Figure 2. The shape parameter on the left figure is $\tilde{\beta} = (p = 1.25; q = 1.55; s = 1.85)$ and on the right figure is $\tilde{\beta} = (p = 2.50; q = 2.75; s := 2.80)$. Note that the vertical axis indicates the fuzzy membership μ of the values on the horizontal axis. The first TFN is symmetrical in shape while the second one is nonsymmetrical. These triangular fuzzy numbers are used to generate their's respective number of failures in the subsequent figures.

4. Conclusion

In this paper we give an example on how to accomodate the fuzziness of a shape parameter in Weibull hazard fuction. We study two different methods. The first one assumes that the fuzziness of the shape parameter propagates to the number of failures with the same form of fuzzy number membership. The second one use the fuzziness of the shape parameter in the computation of the number of failures directly, through the concept of alpha-cut or alpha-level. But in general, we can conclude that using this method the fuzziness of the shape parameter has made the fuzziness of the number of failures increases (in terms of the lenght of its base). Further conclusion that can be derived is that both methods give the same base for the fuzzy number of failures but with different degree of membership.

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