OPTIMAL PREVENTIVE MAINTENANCE OF TWO-PHASE MAINTENANCE POLICY FOR LEASED PRODUCT

Hennie Husniah[1], Andi Cakravastia[2], Bermawi P. Iskandar[3]

[1]Department of Industrial Engineering, Langlangbuana University, Bandung, Indonesia
[2,3]Department of Industrial Engineering, Bandung Institute of Technology, Bandung, Indonesia
[1]hennie.husniah@gmail.com, [3]bermawi@mail.ti.itb.ac.id

ABSTRACT

In this paper, we investigate two dimensional leased contracts for a dump truck operated in a mining industry. To keep the truck in a good operational condition, an imperfect preventive maintenance (PM) policy is applied. When the truck fails then corrective maintenance (CM) is done. PM and/or CM can be leased to an external agent or a lessor for an economic reason. The situation under study is that the lessor offers two dimensional leased contract to the owner of the trucks or the lessee. For repairable leased product, two maintenance models are proposed: (i) maintenance policy of single-phase and (ii) maintenance policy of two-phase. Under these maintenance schemes, the mathematical models of the expected total cost for maintenance policies of single-phase and two-phase are established, the lessor's decision problem has to select the optimal PM degree according to various usage pattern and the operational condition that minimizes the expected total cost. Finally, the features of the optimal maintenance policy are illustrated through numerical examples.

Key words: two dimensional-lease contract, penalty, imperfect preventive maintenance.

1. INTRODUCTION

In an open cut mining industry, loading and transporting mining material are the main processes in which dump trucks play a major role. The cost of these dump trucks can reach up to $ 1 billion. Mean while, the prices of ore, coal and other mining materials have fallen, and this decreases significantly the revenue of mining companies. As a result, the mining companies tend to cut back on capital expense. Leasing dump trucks to an external agent or Original Equipment Manufacturer (OEM) is an alternate way to get a function of dump trucks hauling mining materials.

In most cases, the agent (or OEM) as a lessor gives a lease contract with a full coverage of the maintenance actions (Preventive Maintenance (PM) or/and Corrective maintenance (CM)). There are many literature in maintenance lease equipment have been studied. A lease contract in which PM is taken when the failure rate of the lease equipment reaches a certain threshold value is proposed by (Ashgarizadeh & Murthy, 2000). Failure rate reduction method also has been used by (Rinsaka & Sandoh, 2006) to obtain the optimal periodical maintenance policy for lease equipment. The optimal number and degrees of PM associated with CM introduced by (Jackson & Pascual, 2008). Most studies in lease equipment concentrate on determining the optimal PM policy in specified contract period but none of them combine the imperfect repair and preventive maintenance during lease contract period by age and usage parameters which ever occur first. Later (Iskandar et.al, 2014) introduced a two-dimensional lease contract with considered age and usage as contract limitation for mining equipment.

Offering an equipment with a long maintenance lease contract means that the lessor may incur a greater maintenance costs for servicing the contract and this is of interest to the lessor for reducing the maintenance costs. Since in mining industry to fullfill the operational target they usually lease more than one heavy equipment. Therefore it needs to consider the number of equipment in the lease contract.

In this paper, we extend the work of (Iskandar et.al, 2014) into a two dimentional lease contract for a fleet of lease dump trucks used in a mining industry. The optimal PM (number and degree of PM) is obtained, which minimises the total cost for the lessor. This paper is composed as follows. Section 1 and 2 deal with background and model formulation for the the single-phase and two-phase lease contracts studied. Sections 3 and 4 give model analysis to obtain the optimal number of preventive maintenance and the lessor optimal maintenance level. In Section 5, we give numerical example to illustrate the model and finally we conclude with topics for further research.

2. MODEL FORMULATION

2.1. Notation

The following notation will be used in model formulation.

$\Omega_T = [0,\Gamma_0)\times[0,U_0)$:Lease contract coverage
Δ_y :Preventive maintenance level
X_i :Downtime caused by the *i*-h failure and waiting time
D(t) :Total downtime in (*0,t*]
F(t) :Distribution function of downtime
$\mathfrak{T}$:Down time target
Y :Usage rate
C_r :Repair cost
C₀ :Preventive maintenance cost
C_v :Degree of PM
$\mathcal{C}_{\mathcal{P}}$:Penalty cost per unit of time
J :Expected lease contract cost
$r_y(t)$, $R_y(t)$:Hazard, and Cumulative hazard functions associated with $F(t,\alpha_y)$

2.2. Maintenance Lease Contract for single-phase

We consider that a mining company operates a number of lease dump trucks. The dump trucks is offered with a two-dimensional lease contract with the lease characterised by a rectangle region $\Omega_\Gamma = [0,\Gamma_0)\times[0,U_0)$ where *Γ₀* and *U₀* are the time and the usage limits (e.g. the maximum coverage for Γ_0(e.g. 1 year) or U_0 (e.g. 50.000 km), and hence the lease contract is characterised by a rectangle region Ω_Γ (see Figure 1). All failures under lease contract are rectified at no cost to the lessee. For a given usage rate *y* of the dump truck, the lease contract ceases at $\Gamma_y = \Gamma_0$ for $y \le U_0/\Gamma_0$, or $\Gamma_y = U/y$ for, whichever occurs first. We consider that the lease contract given by the lessor also covers PM action, and hence, during the lease period CM and PM actions are done by the lessor without any charge to the lessee (See Figure 1).

As the lease contract is full coverage (PM and CM), then a penalty cost incurs the lessor if the actual down time falls above the target$(\mathfrak{T})$. If $\mathcal{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the lessor should pay a penalty cost when $\mathcal{D} > \mathfrak{T}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathcal{D} - \mathfrak{T}$. The penalty cost ($\mathcal{C}_{\mathcal{P}}$) is viewed as a penalty given by the lessor. The decision problem for the lessor is to determine the optimal number of PM and degree of maintenance level such that to minimize the expected cost.

We use the one dimensional approach by Iskandar, et.al (2013) and hence it needs to model the expected cost for a given usage rate y. Let $h_y(t) \ge 0$ be the conditional hazard function for the time to first failure for a given y. It is is a non-decreasing function of the item age t and y. Furthermore, we consider the case where age, usage and operating condition where the truck is operated as major factors to influence failure. Here, the accelerated failure time (AFT) model is an appropriate model to be used as it allows to incorporate the effect of the three major factors on degradation of the truck. If the distribution function for T_0 is given by F_0 (T, α_0), where α_0 is the scale parameter, then the distribution function for T_y is the same as that for T_0 but with a scale parameter given by

$$\alpha_y = (y_0/y)^\rho \alpha_0 \tag{1}$$

$\rho \ge 1$. Hence, we have $F(t,\alpha_y) = F_0((y_0/y)^\rho t,\alpha_y)$. The hazard and the cumulative hazard functions associated with $F(t,\alpha_y)$ are given by $r_y(t) = f(t,\alpha_y)/(1-F(t,\alpha_y))$ and $R_y(t) = \int_0^t r_y(x)dx$ respectively where $f(t,\alpha_y)$ is the associated density function. If all failures are fixed by minimal repair and repair times are small relative to the mean time between failures, then failures over time occur according to a non-homogeneous Poisson process (NHPP) with intensity function $r_y(t)$. We will use the accelerated failure time (AFT) model to modelling failures, which allows to incorporate the effect of usage rate on degradation of the dump trucks (see Iskandar et.al, 2013).

Let *Y* be a usage rate of the truck. We consider that *Y* varies from customer to customer but is constant for a given customer (or a given equipment). *Y* is a random variable with density function $g(y), 0 \le u < \infty$. Conditional on $Y = y$, the total usage u at age t is given by $u = yt$. Within the lease coverage, a lease contract ends at $\Gamma_y = \Gamma_0$ for given usage rate y. Two cases need to be considered–i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

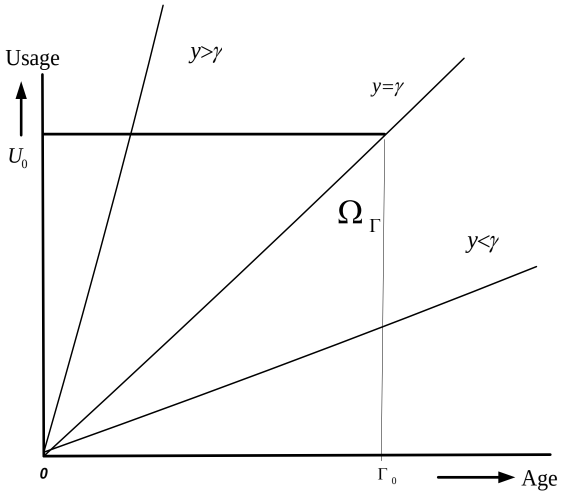

Figure 1. The two-dimensional lease contract

Preventive Maintenance Policy:

We define periodic PM policy for a given $Y = y$. PM policy for a given *y*, is characterised by single parameter τ_y. The equipment is periodically maintained at $k.\tau_y$. Any failure occurring between pm is minimally repaired (See Figure 2). Note $(k+1)\tau_y = T_0$ where k is an integer value.

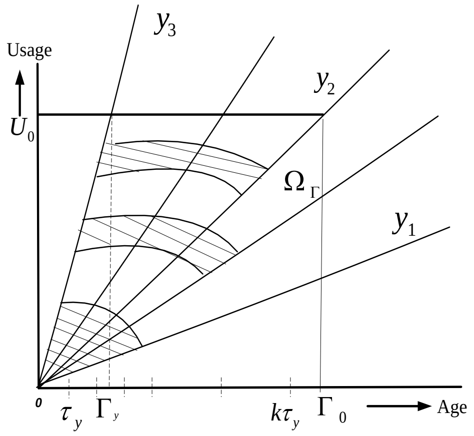

Figure 2. The two-dimensional PM

Modeling of PM effect:

For a given usage rate *y*, the effect of PM actions on the intensity function is given by $r(t_j) = r(t_{j-1}) - \Delta_j$ with $0 \le \Delta_j \le r(t_{j-1}) - \sum_{i=0}^{j} \Delta_i$, Δ_j denotes the reduction of the intensity function after $j^{th}, j \ge 1$, PM action. If the PM action is done at $j^{th}, j \ge 1$ the intensity function is reduced by Δ_j, then for $t_j \le t < t_{j+1}$ the intensity function is given by $r_j(t) = r(t) - \sum_{i=0}^{j} \Delta_i$ with $\Delta_0 = 0$. For simplicity we assume that for each PM action $\Delta_j = \Delta_{j+1} = \Delta$ then $r_j(t) = r(t) - j\Delta$, If any failure occurring between pm is minimally repaired, then expected total number of minimal repairs in $([t_{j-1}, t_j), 1 \le j \le k_y + 1)$ is given by

$$N = \sum_{j=1}^{k_y+1} \int_{t_{j-1}}^{t_j} r_{j-1}(t')dt' = R(\Gamma_0) - \sum_{j=1}^{k_y} \left(\Gamma_0 - jT\right)\Delta_j \qquad (2)$$

For $t_j - t_{j-1} = \tau_y$ then

$$N\left(k_y, \tau_y\right) = R\left(\Gamma_0\right) - \sum_{j=1}^{k_y+1} \left[\left(\Gamma_0 - j\tau_y\right)\right]\left[r\left(j\tau_y\right) - r\left(\left(j-1\right)\tau_y\right)\right] \qquad (3)$$

As the lease contract is full coverage (PM and CM), then a penalty cost incurs the OEM if the actual down time falls above the target $(\mathscr{T})$. If $\mathscr{D}$ is down time (consisting repair time and waiting time) for each failure occuring during the contract, then the OEM should pay a penalty cost when $\mathscr{D} > \mathscr{T}$. The amount of the penalty cost is assumed to be proportional to $\Delta = \mathscr{D} - \mathscr{T}$. The penalty cost ($\mathscr{C}_{\mathscr{P}}$) is viewed as a penalty given by the

OEM. The decision problem for the OEM is to determine the optimal price structure and maintenance level such that to minimize the expected cost.

2.3. Maintenance Lease Contract for two-phase

We consider a two dimensional lease contract where the contract has two limits (or parameters) representing age and usage limits (e.g. the maximum coverage for L (e.g. 1 year) or *K* (e.g. 100.000 km). A typical 2-D lease contract is that a dump truck is leased for *K* (age) or *L* (usage), whichever comes first.

Here, the two-dimensional lease regions form a rectangle region Ω_S (see Figure 3). For $y \le \gamma (= U/W)$ the region is given by $[(\Gamma_0, \Gamma_0 + L) \times (U_y, U_y + K)]$ and $[(\Gamma_y, \Gamma_y + L) \times (U, U + K)]$ for $y > \gamma$ where $\Gamma_y = U/y$ and $U_y = y\Gamma$. The lease contract given by the lessor (OEM) covers all PM and CM to perform both PM and CM (full coverage) for a period of time, L or usage K, whichever occurs first. Let y denote the usage rate of a dump truck. Define γ = L/K. The lease contract terminates due to the age limit, at (K,K/y) if y ≤ γ, and due to the usage limit, at (L/y,L) if y > γ. Here we consider the penalty cost as in section 2.2. The decision problem for the OEM is to determine the optimal PM degree according to various usage pattern and the mining operational condition that minimises the expected total cost.

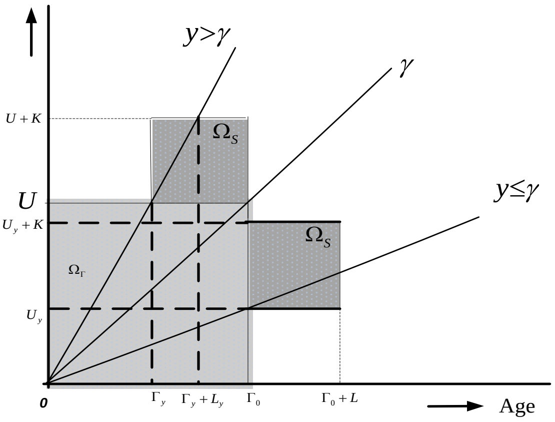

Figure 3. Lease Region Ω_Γ and lease Region Ω_S for $y \le \gamma$ and $y > \gamma$

Preventive Maintenance Policy:

We consider that for a given $Y = y$, PM done by the OEM is an imperfect PM policy. The PM policy for a given *y*, is characterised by single parameter $\tau_y[\nu_y]$ during $\Omega_\Gamma\,[\Omega_S]$. The equipment is periodically maintained at $k.\tau_y$ [$l.\nu_y$]. Any failure occurring between pm is minimally repaired (See Fig. 2). We model the PM policy as on section A.

3. MODEL ANALYSIS

We assume that OEM and the owner have the same attitudes to risk, in order to make the solution reach equilibria.

A. OEM's Decision Problem

Here, the OEM's expected profit depends on two cases–i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

For case (i),

During Ω_Γ, the expected cost is given by

$$E[Cost_y] = E[\text{PM cost}] + E[\text{CM cost}] \tag{4}$$

The expected PM and repair cost conditional on *Y*=*y*, is

$$E(\psi_y) = C_r R_0(0, \Gamma_0) + kC_0 - \sum_{j=1}^{k+1} \left[C_r(\Gamma_0 - j\tau_y) - C_v\right]\left[r_y(j\tau_y) - r_y((j-1)\tau_y)\right] \tag{5}$$

During Ω_S, the expected cost is given by

$$E[\text{Cost}] = E[\text{Incentive}] - E[\text{ Penalty}] - E[\psi_y]$$
where,
$$E[\psi_y] = E[\text{PM cost}] + E[\text{CM cost}] \tag{6}$$

Expected of Penalty Cost:

Let $D(t)$ and ζ denote the sum of down time after a failure (including repair time), and down time target of the equipment in (*0,t*). The expected penalty cost is given by $EP(L) = \mathcal{C}_{\mathcal{P}}\bar{G}(\zeta)N(\ell, \nu_y)$ where $\bar{G}(\zeta) = \int_\zeta^\infty (z - \zeta)g(z)dz$, $\mathcal{C}_{\mathcal{P}}$ is the penalty cost and $N(\ell, \nu_y)$ denotes the expected number of failure in interval $(\Gamma_0, \Gamma_0 + L]$.

Expected Incentive Cost:

The expected of incentive earned in $(\Gamma_0, \Gamma_0 + L]$ is given by

$$EI(L) = C_I \int_0^\zeta G(z)dz \tag{7}$$

Expected of CM cost:
Let C_m is minimal repair cost then the expected repair is given by

$$EC(W,L) = C_m N(\ell, \upsilon_y) \quad (8)$$

where $N(\ell, \nu_y)$ is expected number of failures in $(\Gamma_0, \Gamma_0 + L]$.

Expected PM cost
With cost of ℓ PM is given by $\ell C_0 + C_v \sum_{m=1}^{\ell} \delta_m$ then the expected PM cost is

$$E[PM \text{ cost}] = \ell C_0 - \sum_{m=1}^{l} \left[C_r \left(L - m\nu_y \right) - C_v \right] \delta$$

Where $\delta = [r(m\nu_y) - r((m-1)\nu_y)]$ (9)

For case (ii), the expected cost of the OEM is given by (4) and (6) but it needs to replace Γ_0 with Γ_y and L with L_y.

4. OPTIMAL OPTION

In this section we will look for the optimal value of parameters $k_y^*, \tau_y^*, \Delta_y^*$ by minimizing the total cost function $E_y[\pi]$ subject to constraint $0 \le \Delta_j \le r(t_{j-1}) - \sum_{i=0}^{j} \Delta_i$. The optimal values are obtained involving a two stages. In the first stage, for a fixed k, minimize $E_y[\pi]$ to obtain the optimal values of $\left\{ j\tau_y^*, 1 \le j \le k_y + 1 \right\}$. In the second stage, the optimal k is obtained using the results of the first stage.

5. NUMERICAL EXAMPLE

We consider that $F(t;y)$ the time to the first failure for a given usage rate y is given by the Weibull distribution with $F_y(t;\alpha_y) = 1 - \exp(-t/\alpha_y)^{\beta}$, and its hazard function is $r_y(t) = \beta\left(t^{\beta-1}/(\alpha_y)^{\beta}\right)$ where α_y as in (1). The other parameter values be as follows. Β = 2.5, Γ_0 = 24 (months), L = 24 (months), U=24 (1x10^{4}Km), K = 24 (1x10^{4}Km) (γ = U/W = 1), y_0 = 1, and $C_v = 0.5C_m$, ζ = 80 (hours) or 4 (days) or, $\mathcal{C}_{\mathcal{P}} = 3K$. The down time distribution is given by the Weibull distribution with α = 1, β = 0.5. Other parameter values are given in Table 1.

Table 1. Cost Parameter Values

Param	K	C_b	C_0	C_m	$\mathcal{C}_{\mathcal{P}}$	C_i
Value (10^3	0.5	16.87	0.05	0.1	3K	$\frac{\mathcal{C}_{\mathcal{P}}}{2}$

*Assuming that equipment operates 2025 hours/year.

Table 2 shows results for ρ = 1.2 and 2.0 coresponding to high incline and very hilly, respectively. For a given y, and α_0 (or reliability level), the optimal expected cost increases as the usage rate y increases. This is as expected since the increasing in y causes the failure rate to increase. And this in turn increases the number of failures under warranty, which requires more frequent (k_y*) PM and a higher δ^{y^*} to minimize the maintenance cost.

It is seen that under service contract coverage, larger values of the usage rate result in shorter periods of time between PM actions (ν_y*), which means that the reliability of the equipment has been decreased. As a result, the improvement factor is increased and a larger pricing should be negotiated as well to get a win win solution. As a result the average price of service contract (P_G^*) decreases with the increasing in y, since the penalty cost increases when the n umber of failures increases. We also observe the same behavior as the the operating condition is more severe (ρ is bigger), since the reliability of equipments gets worse as long as the unit deteriorates rapidly with time.

6. CONCLUSION

We have studied a two dimensional service lease contracts for a dump trucks with downtime as performance measures and incentive. The decision problems for both the owner and OEM are obtained

(i) the optimal level maintenance for the owner, and
(ii) the optimal price for the OEM.

In this paper, every failure is modeled by one dimensional approach. One can consider two dimensional approach i.e. a bivariate failure distribution.

7. ACKNOWLEDGMENT

This work is partly funded by the Ministry of Research, Technology, and Higher Education of the Republic of Indonesia through the scheme of “Desentralisasi 2016”.

8. REFERENCES

(a) E. Ashgarizadeh, and D.N.P. Murthy, (2000). “Service contracts,” *Mathematical and Computer Modelling*, vol. 31, pp. 11–20.

(b) K. Rinsaka, and H. Sandoh, (2006). “A stochastic model on an additional warranty service contract,” *Computers and Mathematics with Applications*, vol. 51, pp. 179–188.

(c) C. Jackson, and R. Pascual, (2008). “Optimal maintenance service contract negotiation with aging equipment,” *European Journal of Operational Research*, vol. 189, pp. 387–398, 2008.

(d) Iskandar, B.P., Husniah, H. and Pasaribu, U.S. (2014). “Maintenance Service Contract for Equipments Sold with Two Dimensional Warranties”. *Journal of Quantitative and Qualitative Management*.

(e) Iskandar, B.P., Pasaribu, U.S. and Husniah, H. (2013). “Performanced Based Lease contracts For Equipment Sold With Two Dimensional Warranties.” *Proc. of CIE43*, pp.176–183, Hongkong.

(f) Iskandar, B.P., Pasaribu, U.S., Cakravastia, A., and Husniah, H. (2014b). “Performance-based Lease contract for a fleet of dump trucks used in mining industry.” *Proc. of TIME-E*, Bandung.

(g) Husniah, H, Pasaribu, U.S., Cakravastia and Iskandar, B.P*.,* (2014). *“*Performance-based Lease contract for aequipment used in mining industry.” *Proc. of ICMIT*, Singapore.

(h) Iskandar, B.P., Murthy, D.N.P. and Jack, N. (2005). “A new repair-replace strategy for items sold with a two dimensional warranty”. *Comp. and Oper. Research*, 32(3),669–628.

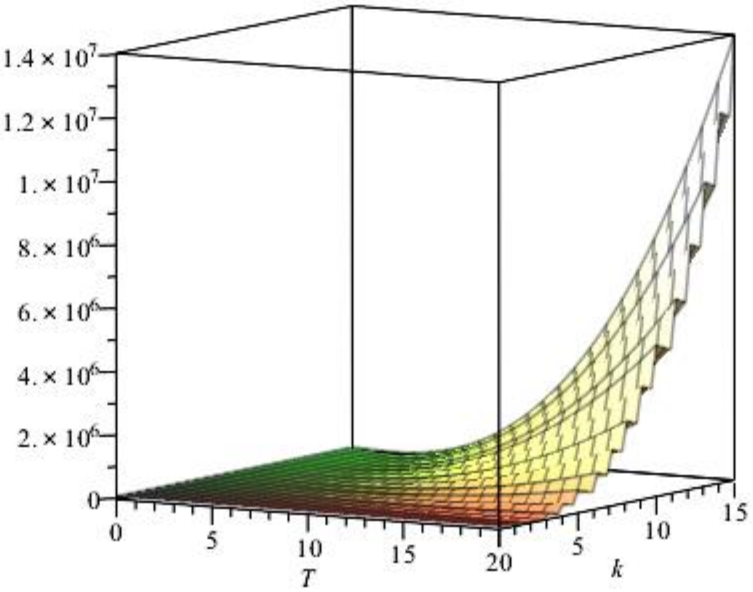

Figure 5. Expected Cost in Single-phase Ω_{Γ}

Table 2. Results for Lease Contract with C_r = 100, ρ = 2, ζ = 80 hours and α_0 = 4 months

Single-Phase $\Omega_\Gamma = \Omega_s$									
Ω_Γ					Ω_s				
$\bar{y}$	k_y^*	τ_y^*	δ^{y*}	$E_y\left[C_w\right]$	ℓ_y^*	v_y^*	δ^{y*}	$E_y\left[\pi_s\right]$	Total Lease Cost
Low Usage Rate									
0.6	3	6.23	0.17	374.28	3	6.23	0.17	11917.83	12292.11
0.8	7	3.01	0.24	911.52	7	3.02	0.25	11531.60	12443.12
1.0	13	1.70	0.32	1834.08	13	1.71	0.32	10753.70	12587.78
High Usage Rate									
1.2	21	0.89	0.47	3551.01	21	0.95	0.52	7082.42	10633.43
1.4	1	10.12	57.39	8107.I5	1	5.65	0.81	5378.36	13485.54