

A probabilistic study on the geometrical design of gravity retaining walls

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Abstract

Purpose – The purpose of this study is to introduce a relatively simple method of probabilistic analysis on the dimensions of gravity retaining walls which might lead to a more accurate understanding of failure. Considering the wall geometries in the case of allowable stress design, the probability of wall failure is not clearly defined. The available factor of safety may or may not be sufficient for the designed structure because of the inherent uncertainties in the geotechnical parameters. Moreover, two cases of correlated and uncorrelated geotechnical variables are considered to show how they affect the results.

Design/methodology/approach – This study is based on the failure and stability of gravity retaining walls which can be stated in three different modes of sliding, overturning and the foundation-bearing capacity failure. Each of these modes of failure might occur separately or simultaneously with a corresponding probability. Monte Carlo simulation and Taylor series method as two conventional methods of probability analysis are implemented, and the results of an assumed example are calculated and compared together.

Findings – The probability analysis of the failure in each mode is calculated separately and a global failure mode is introduced as the occurrence of three modes of sliding, overturning and foundation-bearing capacity failure. Results revealed that the global mode of failure can be used along with the allowable stress design to show the probability of the worst failure condition. Considering the performance and serviceability level of the retaining structure, the global failure mode can be used. Furthermore, the correlation of geotechnical variables seems to be relatively more dominant on the probability of global failure comparing to each mode of failure.

Originality/value – The introduced terminology of global mode of failure can be used to provide more information and confidence about the design of retaining structures. The resulted graphs maintain a thorough insight to choose the right dimensions based on the required level of safety.

Keywords Monte Carlo simulation, Probabilistic analysis, Probability of failure, Retaining wall analysis, Taylor series

Paper type Research paper

1. Introduction

Retaining cantilever walls, usually made of concrete or filled with compacted geomaterials, are among the most frequently designed retaining structures. Design of these structures must satisfy the equilibrium state while having enough stability against exerted bending moments and shear stresses along with the ability of providing necessary slight movements for earth pressures to be mobilized behind it (Peck and Hanson, 1974). However, providing sufficient stability for retaining structures depends on the level of performance and serviceability in which it can be crucial to have no displacement for some cases. The probability of failure can be considered to clarify the boundaries that the design case is expected to be safely working.

Conventional methods of retaining walls design are based on deterministic assumed parameters for soil and deals with safety factors (SFs) to estimate the equilibrium state of the wall or check if the desired values are met in analysis

procedure. In SF-based designs, the soil-structure system is examined for three modes of failure. Stability because of the sliding along foundation base, foundation-bearing capacity and overturning are supposed to be modes of failure which are governing the equilibrium state of the wall (Bowles, 1997; Terzaghi *et al.*, 1996).

By the evaluation of SF in each mode of failure distinctively, in a way that the corresponding value is equal to or larger than a specific limit, system condition of stability can be defined. In most of the design codes and references, it is recommended to construct wall foundation in a depth to provide the adequate resistance against complex layered earth or soil slope (Zevgolli and Bourdeau, 2010). Moreover, load eccentricity from one third of the foundation base is proposed to be checked and calculated, which is playing an important role in the magnitude of bearing capacity in conventional design procedures. Wall settlements must be checked beside the calculations and analysis which is mostly controlled by the type of soil and its density underlying the base (Bowles, 1997; Das, 1992).

All of the above-mentioned criteria are required to be satisfied for an earth retaining structure, while the available formulas and theories are mostly based on simplifications and there are many uncertainties involved in the final achieved

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results. Furthermore, uncertainties in the design loads which are presumed to be acting on the wall lead to inconveniences, and sometimes unfavorable failures might occur against entire precautions. It can be inferred that FS is merely an experience-based factor which cannot be generalized in global engineering knowledge. Using design codes or regulations, similar factor of safety is used in different conditions in which various degrees of uncertainty are involved (Duncan, 2000). However, factor of safety is being used in most of the contemporary projects and corresponding methods of analysis are known as allowable stress design (ASD).

Accordingly, reliability-based methods are proposed to consider marginal safety which can be defined based on the performance of the geotechnical structures. Researchers have reported examples for reliability use in geotechnical engineering for estimating the uncertainty involved in soil strength and the corresponding parameters (Benmeddour et al., 2012; Christian et al., 1994; GuhaRay and Baidya, 2015, 2016; Kulhawy, 1993; Lacasse and Nadim, 1996; Lumb, 1966; Phoon and Kulhawy, 1999; Tang et al., 1999; TH, 1974; Vanmarcke, 1977; Wang et al., 2016; Zhang et al., 2014). Design methods consider the geotechnical variability and are known as Load and Resistance Factor Design (LRFD) in the USA and Limit States Design (LSD) in Canada and Europe (Basheer and Najjar, 1998; Cardoso and Fernandes, 2001; Orr, 2000; Phoon et al., 2003). Another imperfection of conventional design methods is assuming partial evaluation of the SF according to the particular failure modes (Zevgolis and Bourdeau, 2010). Moreover, probability-based analysis deals with most of the drawbacks involved in the conventional design methods (Wang, 2013; Zevgolis and Bourdeau, 2010).

These methods are also of the popular approaches which are being used because of the current advances of processing technology. Massive required calculations for data simulation and realizations are able to be done in a fraction of time. In probabilistic analysis, uncertainty involved in each design parameter is assumed and categorically considered for the calculation of the reliability. Furthermore, the probability of failure occurrence for each mode and the general mode of failure can be investigated explicitly (Wu, 2015; Zevgolis and Bourdeau, 2010). The probability of failure for each mode can provide an acceptable performance-based design for geotechnical structures such as gravity retaining walls.

The objective of this study is to introduce a relatively simple method of probabilistic analysis on the dimensions of gravity retaining walls which might lead to a more accurate understanding of failure. Moreover, it is aimed to maintain a comparative study between two popular methods of probability design, Taylor series reliability analysis as described by Duncan (2000) and Monte Carlo (MC) simulation algorithm (Wang, 2013; Zevgolis and Bourdeau, 2010). Considering the wall geometries in the case of allowable stress design, the probability of wall failure cannot be clearly defined and the available factor of safety may or may not be sufficient for the designed structure because of the inherent uncertainties in the geotechnical parameters. Furthermore, the effect of correlation between geotechnical variables on the probability of failure is another purpose of this study which is discussed later.

2. Methodology

2.1 Failure and stability terminology

Backfilled gravity retaining walls are one of the most commonly designed geotechnical structures. Geometrical parameters are considered as shown in Figure 1.

Herein, the static loading condition is considered and the investigation of failure concerns with overturning (OT) around the toe, base sliding (SL) and the foundation-bearing capacity (BC). These three failure modes (instability conditions) are depicted schematically in Figure 2.

It should be noted that if it is supposed to deal with special structures such as bridge support structures, walls built on inclined ground and/or when layered soil is beneath the structure, then stability in the depth is necessary to be considered and should be included in the modeling of soil. In the present work, it is assumed that the wall is constructed on a ground with no stratification and the settlement analysis is not performed.

In reliability-based analysis of backfilled gravity retaining wall, modes of structural failure needs to be inspected to clarify the internal instability conditions. Structural stability is not mentioned in this study and the focus is on the uncertainties in geotechnical parameters. In addition, external stability and corresponding modes of failure are extracted. Generally, the internal stability of retaining wall would not face failure because of high resistance of the reinforced

Figure 1 Schematic illustration of the gravity retaining wall parameters addressed in this paper

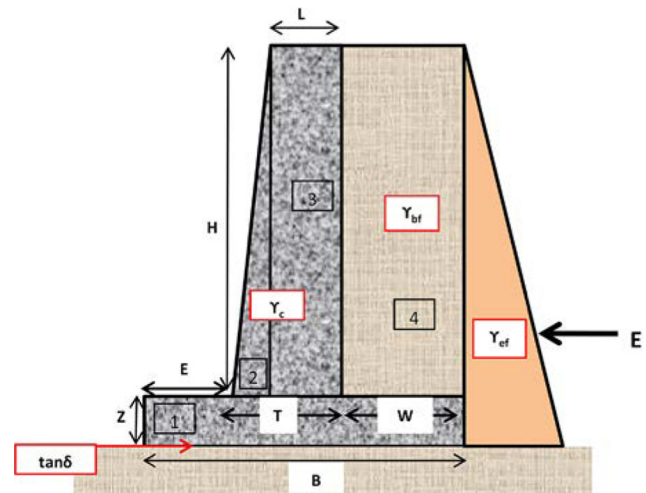
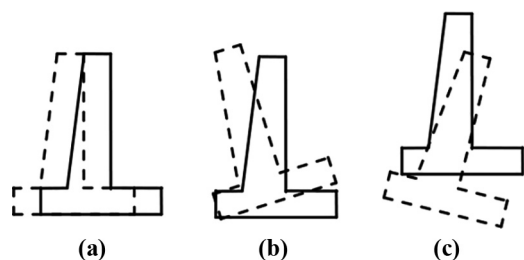


Figure 2 Modes of failure because of (a) sliding, (b) overturning and (c) bearing capacity instability



concrete which is less vulnerable to uncertainties in strength parameters comparing with geotechnical parameters. In external modes of failure, uncertainties in soil shear strength parameters effect on both resistance and load. Thus, the assumption here is that the structural components are performing well and internal structural stability is provided.

It must be explained that failure and the probability of its occurrence is not a catastrophic collapse. For example, sliding of retaining wall, which is considered as a failure mode, would not be disastrous. In case that the wall moves smoothly a small distance away from the backfill, the pressure from the earthen material (soil, rock, etc.) on the wall would be reduced and no further sliding would be occurred. Afterward, in case of increase in earth pressure, because of creep in backfill soil, sliding might be taken place again. Finally, if sliding occurs frequently, then it might result in important displacement of the structure which could amount to unacceptable performance, but not disastrous collapse (Duncan, 2000; Leonards, 1982). Thus, it is expected to keep a clearer meaning as “Inadequate Serviceability” in mind.

2.2 Conventional stability formulation: deterministic approach

According to conventional geotechnical design procedures for this type of wall, the mass of backfill overlaying behind the wall is assumed to form a resisting block attached to the structure (Peck and Hanson, 1974). Active earth thrust is exerting on a hypothetical interface between this resisting mass and the retained backfill is computed according to Rankine’s theory. For dry backfill conditions, and in addition to the self-weight of the wall (W_{Conc}) and the weight of the soil above the base (W_{Soil}), the earth pressures are applied as well. The lateral force (active) earth pressure P_A acting on the back of the wall is as follow:

$$P_A = \frac{1}{2}K_A\gamma_{ef}H^2 - 2c\sqrt{K_A}H \tag{1}$$

in which K_A is the coefficient of active earth pressure, γ_{ef} is the unit weight of retained soil, H' is the total height of the wall ($H+Z$) and c is the cohesive intercept of the soil. In Figure 1, two types of soil are shown to be behind the wall, backfill soil and the retained soil. In the probability analysis in this study, both are presumed to be made from the same material, while they can be distinct in general cases. Horizontal active earth pressure coefficient (K_A) for backfill and vertically retained soil behind the wall is given by:

$$K_A = \frac{1 - \sin \phi}{1 + \sin \phi} \tag{2}$$

in which ϕ is the soil friction angle. Moreover, a passive force is acting on the front side of the wall if there is an embedment depth (D) for wall base. This force is not usually considered because of the conservations. The soil pressure q that acts vertically on the base of the wall has a maximum and minimum value as:

$$q_{\max, \min} = \frac{\sum V}{B}(1 \pm 6e/B) \tag{3}$$

where, $\sum V$ is the summation of vertical forces acting on the wall, B the width of the base of the wall and e is the eccentricity of the loads resultant with respect to the centerline of the base given by:

$$e = \frac{B}{2} - \frac{\sum M_R - \sum M_{OT}}{\sum V} \tag{4}$$

where $\sum M_R$ and $\sum M_{OT}$ are the summations of overturning resisting and driving moments, with respect to the toe of the base. Typically, the resultant is required to intersect the base of the wall within the middle third; hence, the entire area beneath the base is theoretically subjected to compression (Peck and Hanson, 1974). Numerically, this means that the eccentricity must be smaller than or equal to one sixth of the base length. If the resultant falls on the right side of the base centerline, then based on Equation (4), the eccentricity becomes negative. So the condition that must be satisfied is as follow:

$$e \leq \frac{B}{2} \tag{5}$$

The shear resistance denoted by S that is acting along the interface between the base of the wall and the foundation soil. Typically, this is given by:

$$S = B(c') + \sum V.\tan \delta \tag{6}$$

in which c' is the cohesive intercept along the interface and δ is the interface friction angle along the wall and the soil at the base. c' and δ are supposed to be $0.7c$ and 0.7ϕ , respectively, in this study.

In principle, the wall is safe when the loads that tend to activate a mechanism of instability, are smaller than or equal to the loads that tend to resist to this mechanism (capacity–demand model) (Bowles, 1997; Das, 1992; Zevgolis and Bourdeau, 2010). In a traditional deterministic analysis based on ASD, a SF would be computed for each modes of failure using nominal values of the controlling parameters. Each of the resulting SFs would then be required to be greater than a recommended empirical value, specific to the particular failure mode. Required SFs are typically in the order of 1.5-2 for base sliding, 1.25-2 for overturning and 2-3 for bearing capacity (Bowles, 1997; Terzaghi et al., 1996). The fact that these required values are larger than one is an important uncertainty being presented in the design process, which is going to be discussed and shown to be improved in probability design procedure.

2.3 Failure modes

For each mode of failure, a performance function can be assigned with which equilibrium of the structure (external stability) can be defined. These functions can have the same definition as factor of safety and entitled failure ratios (U_i) in this study. Failure ratios with respect to sliding, overturning and bearing capacity inadequacy are U_{SL} , U_{OT} and U_{BC} respectively, and expressed by:

$$U_{SL} = \frac{\sum P_R}{\sum E_{SL}} = \frac{S}{P_A} \quad (7)$$

$$U_{OT} = \frac{\sum M_R}{\sum M_{OT}} \quad (8)$$

$$U_{BC} = \frac{q_u}{q_{max}} \quad (9)$$

Based on the defined performance functions, failure occurred when the related U_i is less than one. The probability $P_{Fi} = P[U_i \leq 1]$ for any given mechanism i which are SL , OT and BC . Global system stability aside from modal stability can be studied further if the following expression can be met:

$$P_{F,sys} = P[(U_{SL} \geq 1) \cap (U_{OT} \geq 1) \cap (U_{BC} \geq 1)] \quad (10)$$

In system stability analysis, all three modes of failure need to be equal or greater than one; otherwise, the global stability will not occur. At the end of this paper, system stability is intended to be discussed.

2.4 Taylor series reliability analysis (TS)

As it was noted previously, there are many sources of uncertainties involved in a gravity retaining wall design which needs to be considered in case that an acceptable performing structure is expected. Taylor series method is one of the probability-based analysis methods. The steps which must be followed for using the Taylor series method are presented through (1) to (4) below.

- 1 Determination of most likely value (MLV) (usually are assumed to be the same as the mean of input parameters) of entire parameters in the analysis and evaluating the factor of safety based on the deterministic methods. This is F_{MLV} and can be determined in the present study by using Equations (7) to (9), in which instead of U_i , F_{MLV} is substituted. This provides a simple way to compare evaluations with each other.
- 2 Estimating the standard deviations (σ) of the parameters that involve uncertainty, which can be followed by acquiring the standard deviation for a number of input parameters or can be extracted out of literature for each individual property.
- 3 Computing the factor of safety while for each parameter an increased amount of a standard deviation (Plus Value or F^+) and then a decreased amount of one standard deviation (Minus Value or F^-) comparing with its most likely value (MLV) with no change in other parameters (keep them equal to most likely values) is applied. This would require $2N$ computation, where N is the total number of considered parameters whose quantities are being changed in the analysis. In addition, N times for each of F^+ and F^- states calculations are performed. ΔF can be evaluated using F^+ and F^- , for each input parameter in which uncertainties are involved. The standard deviation (σ_F) and coefficient of variation for the

factor of safety (V_F) can be obtained using following equations.

$$\sigma_F^i = \sqrt{\sum_{j=1}^N \left(\frac{\Delta F_j}{2}\right)^2} \quad (11)$$

$$V_F^i = \frac{\sigma_F^i}{F_{MLV}^i} \quad (12)$$

where j is the coordinator varies from 1 to N for each individual input parameter.

- 4 Finally, with the value of F_{MLV}^i from (1) and the value of V_F^i from (3), determining the value of P_{Fi} is possible based on a desired distribution. In fact, P_{Fi} is the probability of failure in each mode and can be calculated using the reliability index (β_i) as follow:

$$P_{Fi} = 1 - F_X[\beta_i] \quad (13)$$

where $F_X[-]$ is the cumulative distribution function (CDF) for a distribution. Reliability index is introduced to be $\beta_i = (F_{MLV}^i - 1)/(\sigma_F^i)$ for normally distributed and $\beta_i = \ln [F_{MLV}^i / \sqrt{1 + V_F^i}] / \sqrt{\ln(1 + V_F^i)}$ when log-normally distributed input variables are governing the problem. For simplicity purposes, input parameters of this study are assumed to be normally distributed, though the general validity is not loss and other distributions can be replaced if required.

2.5 Monte Carlo simulations

Using the algorithm of MC simulation, one is able to generate as many as required random variables according to the target statistical properties by using an appropriate distribution. Normal distributed random parameters are generated and required analysis is performed using EXCEL spreadsheet. As a deterministic point of reference, the mean values of the failure ratios (U_i) are computed, which are comparable with F_{MLV}^i . This is so because when the trials are large enough, the mean values are tend to be equal with the expected values (Harr, 1987). For the probabilistic analysis based on MC algorithm simulations, 30,000 realizations are performed. Number of realizations is reasonable to keep errors in the computed probabilities within acceptable limits (between 0.005 and 0.01) according to error estimation procedure presented in Baecher and Christian (2003). The probability of failure is then given by:

$$P_{Fi} = \frac{n_{Fi}}{N} \quad (14)$$

Herein, N is the number of realizations and n_{Fi} is the number of times that U_i is less than one. Steps of this probabilistic method can be find in many books (Baecher and Christian, 2003).

2.5.1 Input parameter cross-correlation

The effect of cross-correlation between the involved random variables, particularly between friction angle, cohesive intercept and soil unit weight, is considered to be evaluated in this paper. Coefficient of correlation can be defined as:

Table I Used data in probabilistic analyses

Geometrical inputs	Geotechnical variables			
	Parameter	SD (σ)	Mean value (μ)	
$B(m)$	3.6	$\tan(\delta)$	0.07	$\tan(0.7 \times \varphi)$
$Z(m)$	0.5	$\gamma_{ef}(kN/m^3)$	2	18
$E(m)$	0.3	$\gamma_{bf}(kN/m^3)$	2	18
$T(m)$	0.3	$\gamma_c(kN/m^3)$	5	23.5
$W(m)$	3	c (kPa)	5	10
$H(m)$	9	$\varphi_{soil}(Deg)$	5	30
$L(m)$	0.3	$\rho_{\theta,c}$	–	–0.6
$D(m)$	0.5	$\rho_{\phi,\gamma}$	–	0.7
		$\rho_{c,\gamma}$	–	0.1

$$\rho_{X,Y} = \frac{Cov(X, Y)}{\sigma_X \sigma_Y} \quad (15)$$

where $Cov(X, Y)$ is the covariance between two random variables and σ_X and σ_Y are correspond to standard deviations. To generate cross-correlated random variables, random generated numbers must be transformed using a transformation matrix as follow:

$$[R_c] = [R][T] \quad (15.a)$$

$$[T]^T[T] = [\rho] \quad (15.b)$$

If coefficient correlation matrix $[\rho]$ is then assembled, using Cholesky decomposition procedure transformation matrix $[T]$ can be obtained (Press *et al.*, 1992). Thus, this method can be readily used to generate cross-correlated random variables. Values varying in both negative and positive range have been quoted by several authors in literature (Cherubini, 2000; Fenton and Griffiths, 2003; Miller, 2011; Shahin and Cheung, 2011; Wu, 2015). Chosen coefficients are presented in Table I. In case of considering $\rho_{X,Y} = 0$ the pair of X, Y input random variables become statistically independent and uncorrelated random numbers would be generated in the simulation. This state is tested and results are stored to be compared with correlated ones beside Taylor series probability outcomes.

3. Problem definition

For the purpose of comparison between two methods of probability design which discussed previously, the same geometrical and geotechnical input parameters are chosen. Input data used in analysis are presented in Table I.

Statistical properties of gravity wall geometry and geotechnical input variables are used in TS once and for MC two times, which are uncorrelated (MC1) and correlated (MC2) states. After performing these three analyses, W changed increasingly between 1 m and 5 m by 0.25 steps and H between 3 to 9 by 3 m steps. MC realizations are stored to obtain variations of U_i and P_{Fi} according to the changes of wall geometry in both uncorrelated and correlated conditions. The failure ratios and probabilities of failure are then achieved for each mode of stability and for system stability as well through analysis of MC realizations. Further explanations are mentioned in next sections.

4. Probabilistic analysis results

4.1 Comparing results of stability analyses

Results of TS, MC method (uncorrelated and correlated) and deterministic computation for $W = 3m$ and $H = 9m$ are presented and summarized in Table II. As it was mentioned earlier, U_i is a point of reference for providing the comparison between results and can be interpreted as F_{MLV}^i , mean of failure ratios and SF for Taylor series, MC realizations and deterministic methods, respectively.

Based on the data presented in Table II, it can be seen that for sliding mode, approximately equal probability of failure are observed through Taylor series and MC simulations. The value of U_i which is treated as the central value in the analysis process is almost the same for all four categories. It can be stated that cross-correlation does not affect probability of failure in sliding mode.

Although this is not the case for overturning mode of failure, safety ratios are the same as well. $P_{F,OT}$ is nearly doubled for Taylor series method comparing with MC approach. Correlated case is shown to have about 1 per cent less probability of failure comparing to uncorrelated input variables.

For bearing capacity mode, the trend is the same as the overturning and Taylor series probability of failure is roughly two folded of the $P_{F,BC}$ for MC1 and MC2. Moreover, cross-correlation lead the probability of failure to be decreased about four per cent. The effect of correlation observed through more trials and is not shown extremely influences on the probability of failure in sliding and overturning modes.

Meanwhile, the probability of failure variations for bearing capacity are strongly dependent on negative or positive presumed values for soil friction angle and cohesion ($\rho_{\varphi,c}$) rather than two other coefficients. For instance, $\rho_{\varphi,c} = +0.6$ results in nearly 17.86 per cent which is more than that of uncorrelated situation and negative value presented in

Table II TS, MC1 and MC2 results for probabilistic analysis

Method	Modes of failure					
	Sliding		Overturning		Bearing capacity	
	U_{SL}	$P_{F,SL}$ (%)	U_{OT}	$P_{F,OT}$ (%)	U_{BC}	$P_{F,BC}$ (%)
TS	0.932	59.55	1.312	17.44	2.236	26.97
MC1	0.934	57.13	1.313	8.25	2.246	14.51
MC2	0.935	58.83	1.311	7.123	2.235	10.40
Det.	0.932	NA	1.312	NA	2.236	NA

Note: NA: Not achievable

Table II. In addition, in the literature, the same trend is reported for some geotechnical problems, and it is found that correlation between input parameters have a minor effect on the probabilistic results. It is also reported in the previous studies that positive cross-correlations overvalue the failure probability, while negative cross-correlations lead to undervalue probabilities of failure (Fenton and Griffiths, 2003).

4.2 Discussion on Monte Carlo realizations

As mentioned in the preceding sections, values of failure ratios can be extracted out of realizations to be studied for global system stability. Relative frequencies of U_i are illustrated in Figure 3(a)-(c) for the conditions stated in Table I.

Variations of the wall footing length of heel (B) and the height (H) are applied to evaluate the corresponding failure ratios. These are providing the graphs for estimation of U_i according to desired design geometries which can be further compared with the reference deterministic values of SF. Results are presented in Figure 4(a)-(c). Both correlated and uncorrelated values of U_i are plotted which shows that the central values of variables are not varied considerably, while the involved cross-correlation of input parameters is assumed.

Furthermore, the probabilities of each mode of failure are stored while varying the amount of wall foundation and height. These results might be used directly in probabilistic analysis of gravity retaining walls according to prescribed geometry. Figure 5(a)-(c) shows the probability of failures in three modes of stability. Plotting correlated and uncorrelated values together, it can be inferred that cross-correlation lead to minor fluctuations, while the wall base is changed. Generally, increasing the height of the wall provokes the cross-correlated to effect more on the probability of failure.

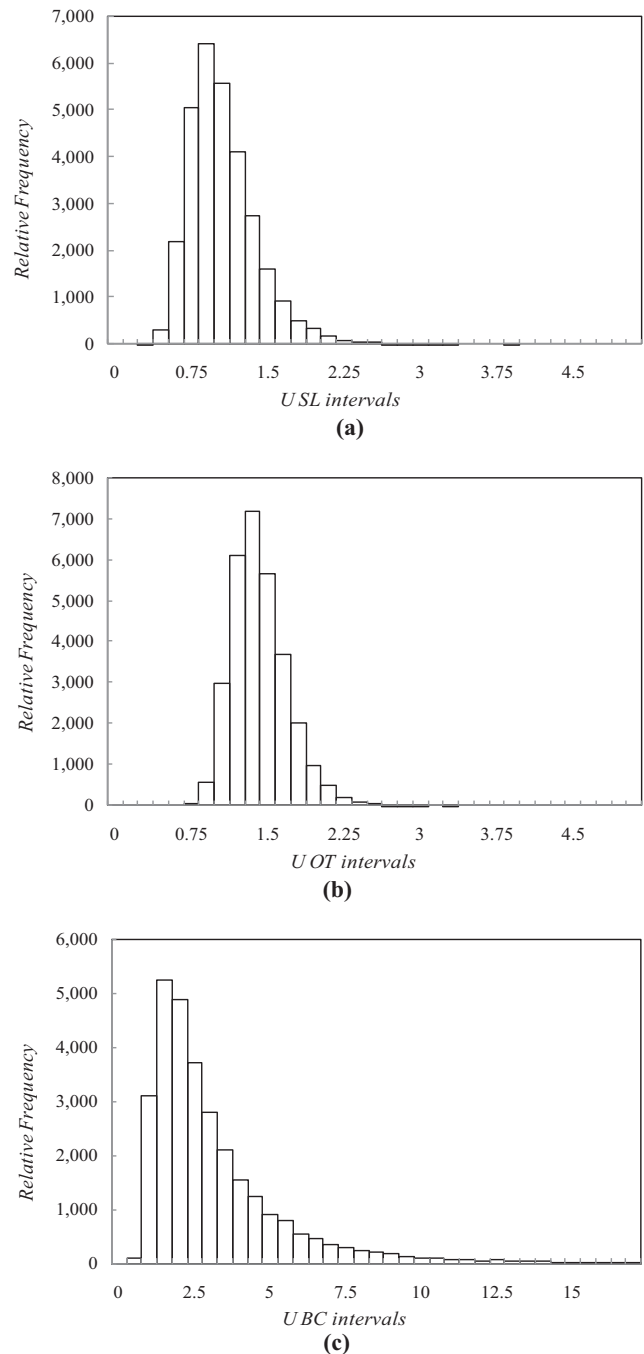
In Figure 5(a), most of the probabilities are approximately laid on zero for wall base dimensions larger than 3 m. However, there is only probability of sliding in this particular geometrical case and relatively larger values are obtained when correlated parameters are considered.

4.3 Global failure mode

According to the flexibility of spreadsheets in data analysis procedure, the probability of global failure which is introduced in Equation (10) can be evaluated. The trend is alike the previously mentioned modal failure analysis, though the definition is changed. Using a conditional state, the probability of system failure can be acquired in a way that it is the number of times where all failure mode functions (U_i) are larger than one divided by the total number of realizations.

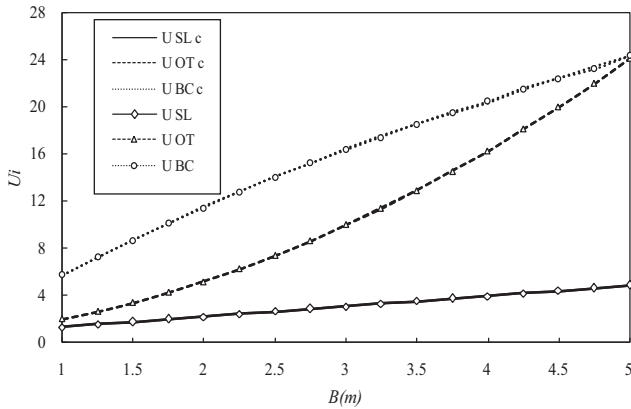
Using the presented graphs, one is able to evaluate the global probability of failure in system which can provide a comprehensive overview of the problem. Variations of $P_{F,SYS}$ with changes in wall base in three sets 3, 6 and 9 m are presented in Figure 6(a)-(c). It can be vividly seen that the cross-correlated values of global probability of failure are approximately larger than the uncorrelated values. In fact, the difference between $P_{F,SYS}$ in correlated and uncorrelated condition increases as the height of the wall decreases. This is so because of small values of $P_{F,SYS}$ for the case that the height of the wall is 3 m which is generally more stable than two other cases.

Figure 3 Relative frequencies of failure ratios (a) U_{SL} , (b) U_{OT} and (c) U_{BC}

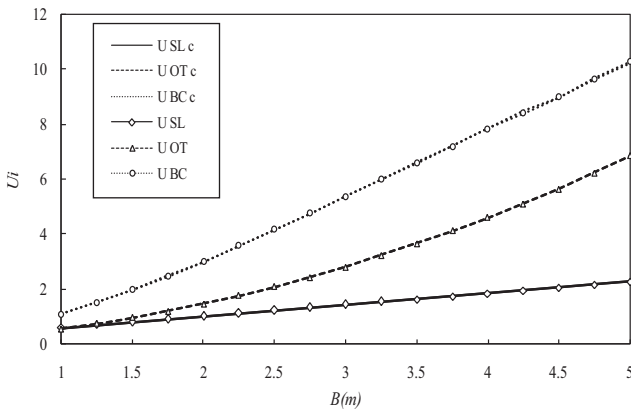


The probability of failure in system decreases with adding to the amount of wall foundation base. The same trend is also reported by Zevgolis and Bourdeau (2010) for changes of system probability of failure along with variations of wall base. However, the wall height directly affects the stability of the wall and in case of increasing; the exerted active pressure is accordingly expected to be raised. Following the increment in wall height, the wall weight would increase. Thus, there is a two-sided equation of equilibrium which needs to be inspected during the analysis.

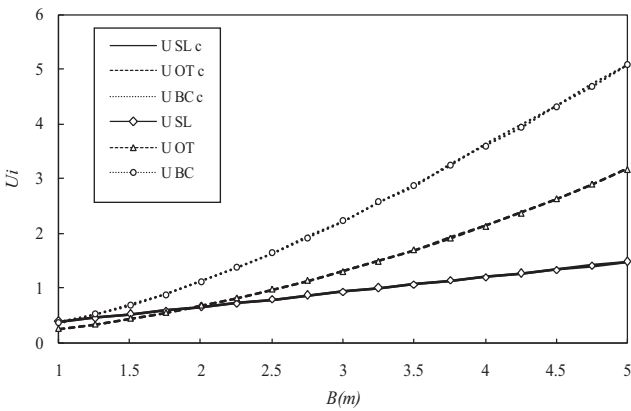
Figure 4 Variations of (a) U_{SL} , (b) U_{OT} and (c) U_{BC} versus wall footing length of heel for 3, 6 and 9 m wall height



(a)



(b)

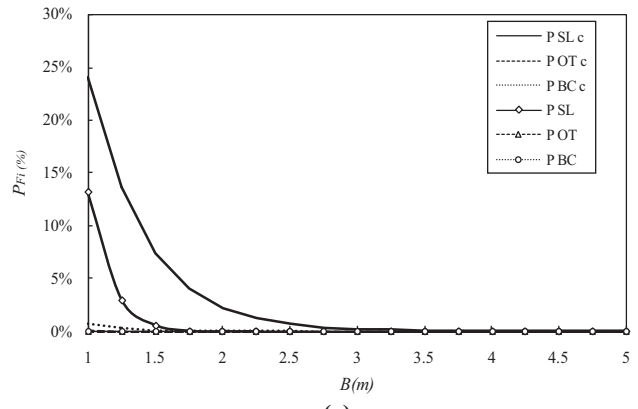


(c)

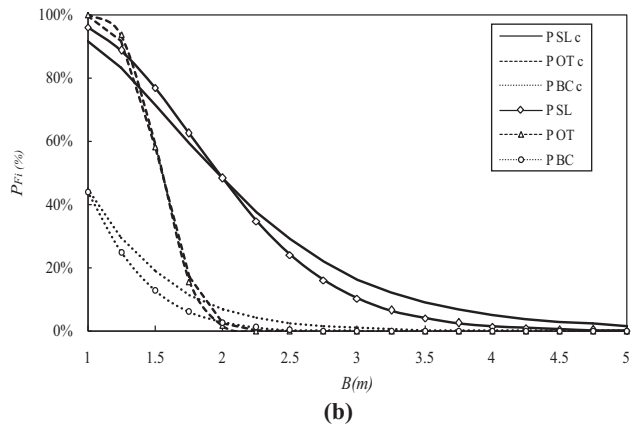
5. Conclusion

There are many uncertainties involved in every geotechnical problem according to the specific conditions of earth materials. There are many variables regarding the soil nature which are fluctuating from one site to another. The exact value of geotechnical parameters cannot be specified all the time and inaccuracies are accompanying the results. Then, using conventional methods of deterministic design

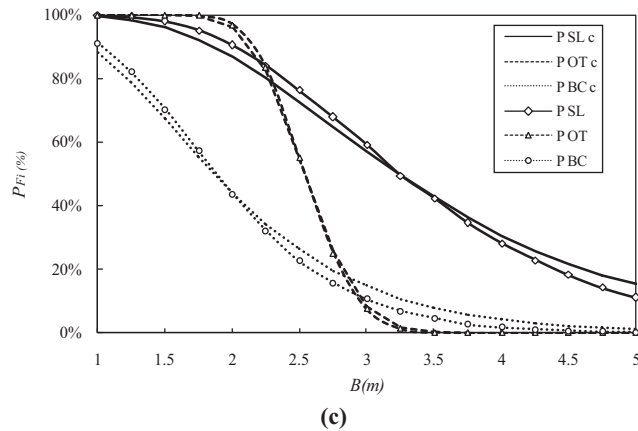
Figure 5 Variations of (a) $P_{F,SL}$, (b) $U_{F,OT}$ and (c) $U_{F,BC}$ versus wall footing length of heel for 3, 6 and 9 m wall height



(a)



(b)

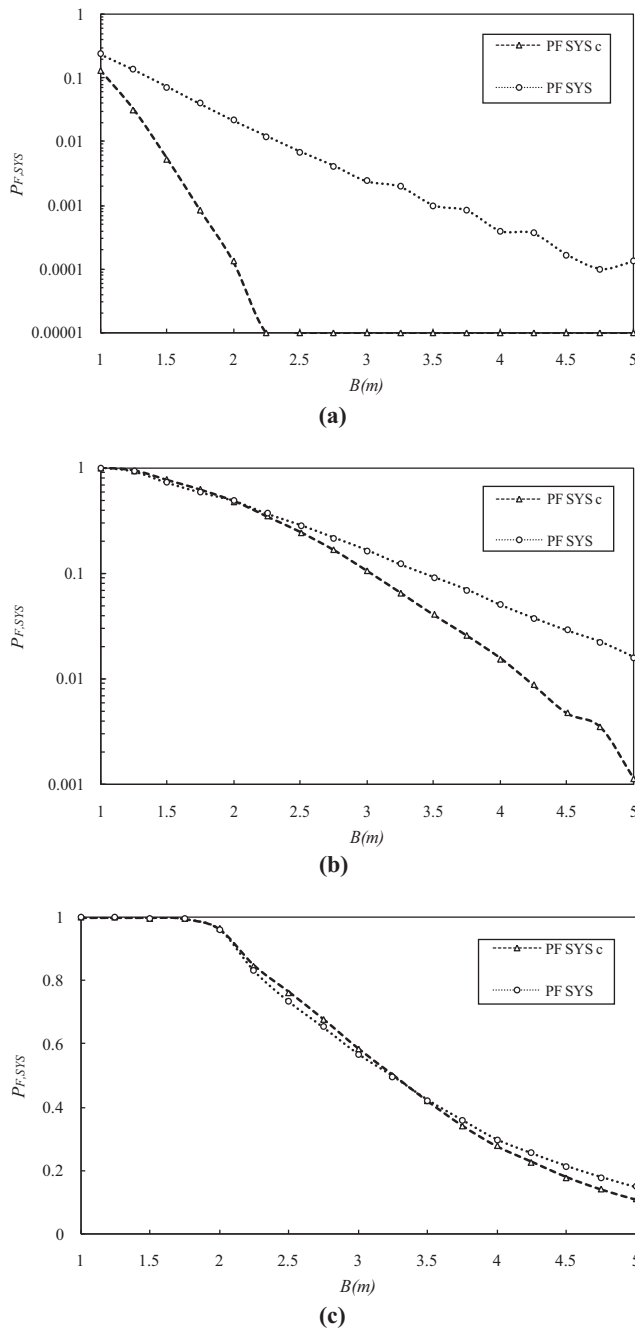


(c)

might encounter with uncertainties as well. Another side of the problem is because of no available general reference such as SFs which are altered from one site of construction to another.

Probabilistic methods can provide a margin for designs which the risk of failure can be specified. Following the require steps for probability of failure calculation, one will be able to determine the reliability. In this paper, a gravity wall is probabilistically analyzed through two methods of Taylor series and MC realizations. Results show that these two methods have the same responses of failure probability in

Figure 6 Variations of $P_{F,SYS}$ versus wall footing length of heel for (a) 3 m, (b) 6 m and (c) 9 m wall height



sliding mode. However, for two other failure modes, the values of Taylor series are two times greater than the calculated probabilities of failure from MC simulations.

In addition, using the realizations which are made based on assumed statistical properties for input parameters, the effect of increasing the wall footing heel length and height on the probability of failure are studied. Modal failure graphs presented considering two cases of cross-correlated and uncorrelated conditions for input variables. Finally, data analysis presented that cross-correlated situation and uncorrelated case approximately lead to the same probability

of failure. Meanwhile, the cross-correlation is influential on the cases with smaller height.

Probability of failure for three modes of sliding, overturning and bearing capacity failure is presented in the text and a global probability of system failure is introduced as well. It is suggested that both analysis are useful to be made to provide a desirable design. Moreover, a performance level must be specified based on engineering judgment to optimize the design and prevent under and overestimations.

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