The Impact of Fleet Size on Remanufactured Product with Usage-based Maintenance Contract

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Abstract- We consider a situation where a mining company operates a number of remanufactured trucks (as a fleet) for transporting mining materials (such as coal, ores) from several mining fields to processing units. A high availability of the fleet of trucks is critical factor for achieving a monthly production target of the company. A usage-based maintenance contracts with coordination and non coordination between two parties is applied to the truck operated in a mining industry. The situation under study is that an agent offers service contract to the owner of the truck after warranty ends. This contract has only a time limit but no usage limit. If the total usage per period exceeds the maximum usage allowed in the contract, then the owner will be charged an additional cost. In general, the agent (Original Equipment Manufacturer/OEM) provides a full coverage of maintenance, which includes PM and CM under the lease contract. The decision problem for the owner is to select the best option offered that fits to its requirement, and the decision problem for the agent is to find the optimal maintenance efforts for a given price of the service option offered. We first find the optimal decisions using coordination scheme and then with non coordination scheme for both parties. A usage-based maintenance contracts with coordination and non coordination between two parties is studied in this paper. The contract is applied to a dump truck operated in a mining industry. The situation under study is that an agent offers service contract to the owner of the truck after warranty ends. This contract has only a time limit but no usage limit. If the total usage per period exceeds the maximum usage allowed in the contract, then the owner will be charged an additional cost. In general, the agent (Original Equipment Manufacturer/OEM) provides a full coverage of maintenance, which includes PM and CM under the lease contract. The decision problem for the owner is to select the best option offered that fits to its requirement, and the decision problem for the agent is to find the optimal maintenance efforts for a given price of the service option offered. We first find the optimal decisions using coordination scheme and then with non coordination scheme for both parties. And also we give numerical examples to show the optimal number of server for the OEM and the optimal strategy for the owner.

Keywords: maintenance contract, two dimensional warranty, fleet, availability, cooperative and non-cooperative game theory

1 INTRODUCTION

Dump trucks are important equipment in an open-pit mining system. The dump trucks are used to load mining materials (such as coal, ores) at a mining site and then tranport the mining material to an unloading site. Usually a mining company operates a number of dump trucks (as a fleet) to fullfil a daily production target. A high availability of the trucks is critical factor for achieving the production target. Preventive maintenance (PM) is an effective way to keep the trucks in a high availability and the PM can be done using age based or conditioned base maintenance. When a truck fails, corrective Maintenance (CM) action is performed, which restores the failed truck to the operational condition.

All dump-trucks operated in a mining industry are sold with a two-dimensional warranty. For example, a

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dump truck is warranted for maximum 3 years or 150.000 km. In order to give full assurance (the dump truck will function as promised over the warranty) to the company (or the owner), the manufacturer offers the warranty and PM in one package and this requires the manufacturer to rectify all failures under warranty as well as carry out PM.

After the warranty expires, the owner is fully responsible to carry out all maintenance (PM and CM) actions for the fleet. The PM and CM actions can be done either in house or by independent agents or the OEM. As dump-trucks and other heavy equipment used in mining sites tend to be complex and expensive, then performing PM and CM actions in house requires expensive maintenance facilities and skilled maintenance crews. As a result, it would not be economical to do PM and CM in house. To sustain high performance of the fleet, the company has to purchase the maintenance services from the original equipment manufacturer (OEM) or an external agent. Often, the OEM is the only maintenance service provider for the dump trucks and offers more than one service contract options (fully, moderate or partial maintenance coverages), and the OEM proactively offers PM and CM to the owner just before the warranty ends.

From the owner's perspective, maintenance programs for the fleet are aimed at not only to sustain high performance (e.g. high availability) for the fleet but also to obtain optimal business profitability. As a result, the owner should select the maintenance contract option that gives high availability of the fleet with reasonable maintenance costs. Whilst the OEM needs toprovide maintenance service to achieve the contracted availability. As the OEM has to perform maintenance services for a group of trucks, then maintanance capacity will affect the service rate and waiting time to get a service. This will in turn influence availability of a truck. In this context, a relevant decision for the OEM is to determine maintanance capacity to attain the the contracted dump truck availability, and the price for each contract option such that to maximise the OEM's profit.

Study of maintenance service contract has received a lot of attention in the literature. Several papers, Murthy and Ashgarizadeh [1], Ashgarizadeh and Muthy [2], and Rinsaka and Sandoh [3] studied maintenance service contract for non repairable items, and formulated decision problems using a Stackelberg game theory with the agent as a leader and owner as the follower. For repairable items, Jackson and Pascual [4], Wang [5] and Wu [11] studied maintenance service contract which involves preventive maintenance policies, and the optimal option is obtained to maximize the expected profit for both the agent and the owner.

In all works on maintenance service contracts discussed above, a penalty cost is modelled based on down time for each failure. In many cases, mining companies consider the availability of dump trucks is a critical performance measure for supporting their business. Hence, a maintenance service contract which ensures a high availability of the equipment at reasonable cost would be an attractive contract for the owner. In Iskandar *et al.* [7] and Iskandar *et al.* [8] , the authors consider maintenance service contracts with availability target for a dump-truck sold with a two-dimensional warranty. Pascual *et al.* [9] developed a model for determining jointly optimal fleet size and maintenance capacity but this work does not deal with maintenance service contracts.

In this paper,we extend the maintenance service contracts studied in Husniah *et al.* [8] to the case of a group of remanufactured trucks (fleet), we propose a new maintenance service contract (MSC) characterized by L representing a time period of contract and $U_{\max}$, representing the maximum usage allowed (See Fig. 1). Here, the contract will not terminate when the usage at time t (<L) is greater than $U_{\max}$, but the owner will be charged some additional cost. The contract will terminate only due to the time limit. This contract can be viewed as the extension of the two dimensional service contract by [1] and [2] where the usage limit is no longer acting as the limit of contract but it is only a maximum usage allowed.

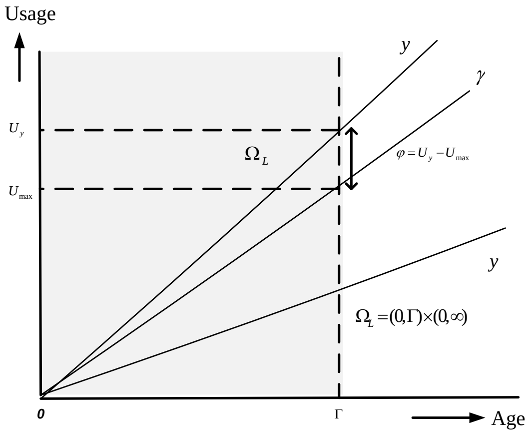

Figure 1. Key elements of maintenance service contract considered

In addition, we study two MSC models based on cooperative and non-cooperative games, where the cooperative case is similar to that studied by [3] but we consider a different maintenance policy (i.e. imperfect PM policy) which fits with the case of dump trucks used in a mining site.

For the cooperative game, it requires two major criteria. First, cooperation should lead to a win-win situation for the owner and OEM, such that the profits of both players become higher compared to independent individual profit [3]. Second, the players should have no incentive to deviate from the non-cooperative solution, i.e., they should modify their profits such that the total maximum solution becomes identical to a Nash equilibrium (for the simultaneous move case) [4].We have allocated the maximum total profit between the players to satisfy the first criterion. In this game, decision makers negotiate over different contract terms .

Next, for the non-cooperative game, we considered: non-cooperative simultaneous move game where the owner and OEM choose their strategies simultaneously. We derive the Nash equilibrium. It is of interest to compare the performance of the non-cooperative contracts and the cooperative alternative. One measure of performance is the difference between the total profit of a non-cooperative contract and that of a cooperative one (which has the maximum total profit).

The remainder of this paper is organised as follows. In section 2 we give model formulation for the service contract studied. The cooperative and the non-cooperative methodologies are described in Section 3. Section 4 analyzes the game solutions, the Nash equilibrium is determined and compares it results with the cooperative solution. Section 5 presents numerical examples to illustrate the MSC models and to investigate their performance. Finally, Section 6 concludes the paper.

In this paper, we extend the maintenance service contracts studied in Iskandar *et al.* [8] to the case of a group of trucks (fleet), and study the contracts where the OEM is more powerful than the owner in the contract negotiation and hence we consider the manufacturer is a leader and the customer as the follower. In addition, the penalty is based on the availability per period (usually one year) and it is assumed that the availability target decreases each year as the equipment deteriorates with usage and age. In addition, this work can be viewed as the extention of maintenance service contracts studied in Murthy and Ashgarizadeh [1] for a repairable case.

This paper is organized as follows. In Sections 2 and 3 we give model formulation, and model analysis. We present numerical examples in Section 4, and finally conclude with topics for further research.

2 MODEL FORMULATION

A. Notation

The following notation will be used in model formulation.

W,U	:Warranty time and usage limits
X_i	:Downtime caused by the *i*- th failure and waiting time
G(x)	:Distribution function of downtime, X_{ji}
ζ	:Total repair time allowed
Υ, L	:Revenue , maintenance contract length
Y	:Usage rate
C_r	:Repair cost done by OEM
C_o	:Preventive maintenance cost per PM
C_v	:Additional cost for level PM created per PM
$\phi_y(O.)$	: Owner profit
$\pi_y(O.)$	: OEM profit
C_b	:The product cost over the contract period
$F(t,\alpha_y)$	:Conditional failure distribution for a given usage rate *y*
$r_y(t),\ R_y(t)$	:Hazard and Cumulative hazard functions associated with *r (t)* and $F(t,\alpha_y)$

B. Warranty Policy

We consider a situation where a mining company operates a number of trucks (as a fleet) for transporting mining materials (such as coal, ores) from several mining fields to processing units. The manufacturer sales each dump truck with a two-dimensional warranty which is characterized by a rectangle region $\Omega_W = [0,W)\times[0,U)$ where *W* and *U* are the time, and the usage limits. All failed trucks under warranty will be fixed by the manufacturer at no cost to the buyer. The expiry of the warranty depends on the usage rate (*y*) of a truck. Hence, for a given usage rate (*y*), the warranty ceases at $W_y = W$ for $y \le U/W$, or $W_y = U/y$, for $y > U/W$. With warranty and PM in one package, manufacturer has to perform PM and CM actions during the warranty without any charge to the owner. The responsibility to do PM and CM actions shifts to the owner once the warranty ends.

C. Maintenance Service Contract (MSC)

The OEM proposes a two dimensional MSC for a period of L (e.g. L =2 year) with a fixed price P_G ,. Here, the MSC has no usage limit or in other words, the contract coverage forms a region $\Omega_S = \left[W_y, L\right)\times\left[yW_y,\infty\right)$ (See Fig. 2). The contract starts at the end of warranty, W_y. However, since the performance of the equipment degrades with age and usage and often the usage contributes more impact to the deterioration of the equipment, then the owner will be charged an additional cost when the usage rate goes beyond the nominal value. This is considered as a compensation to the OEM due to the increase in repair cost.

In general, the OEM provides a full coverage of maintenance, which includes PM and CM, for each MSC dump truck. In other words, all failures under MSC contract are fixed at no cost to the owner and all PM actions are performed without any charge to the owner. It is also stated that the OEM as a service contract provider assures a minimum down time (repair time and waiting time) for each failure and penalty cost incurred when the down time exceeds the predetermined target.

The MSC is defined as follows.

For a price of P_G, the OEM proposes a two dimensional MSC for a period of L. When the total usage (U_y) at L is greater than U_{max}, an additional cost is charged to the owner. The amount of the additional cost is proportional to $\Delta = U_y - U_{\max}$ given by $C_{na}(U_y - U_{\max})$. The additional cost is viewed as a compensation for the OEM as the total usage $> U_{\max}$. And a penalty cost is born to the OEM when the down timeexcess the down time target.Under this option, the OEM agrees to carry out PM and CM in $(W_y, W_y + L]$ or the maintenance service is full coverage.

We study the MSC proposed from the view points of the OEM and the owner. The objective of the OEM is to minimize the expected maintenance cost according to various usage pattern and the mining operational condition whilst the objective of owner is to maximize the expected profit.

In this paper, we consider a situation where a mining company operates a number of trucks (as a fleet) for transporting mining materials (such as coal, ores) from several mining fields to processing units. It is assumed that the OEM has a limited number of maintenance service facilities (or servers) and hence when a truck fails, there would be a chance that the failed truck has to wait before getting a service. To control downtime below the target, the OEM needs to determine the number of service facilities (or servers) in order to minimise waiting time to get a maintenance service. As a result, the decision problem for the OEM is to determine (i) maintenance capacity (the number of service facilities) and (ii) the optimal price structure (i.e. repair cost for option1, the price of a full service contract for option2 and price of a partial service contract for option3) such that to maximize the expected profit.

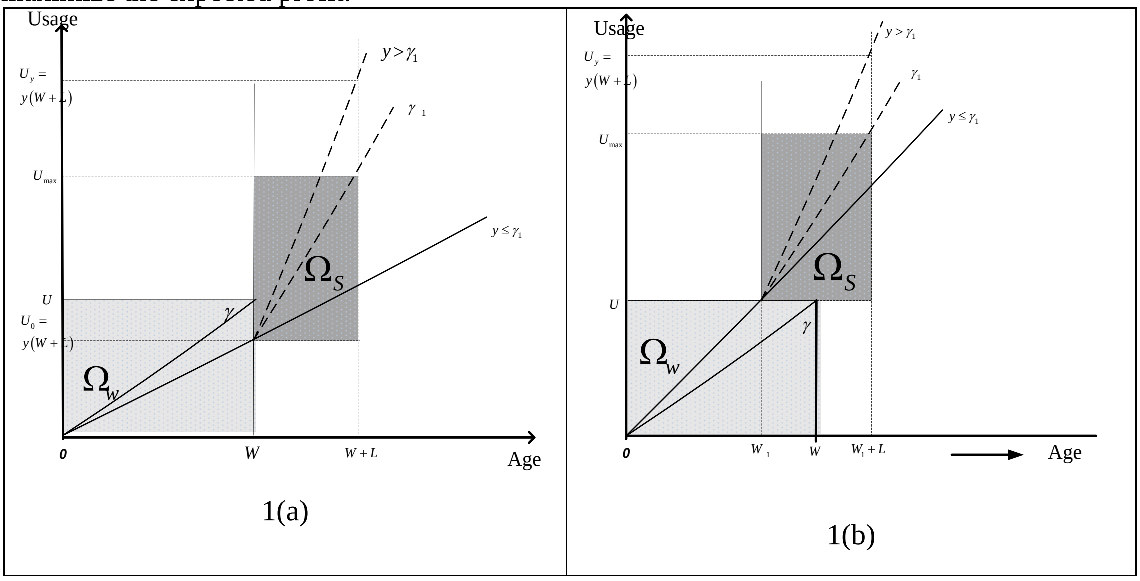

Figure 2.Warranty region Ω_W and *service* contract region Ω_S for (a) $y \le \gamma$ and (b) $y > \gamma$

D. Equipment Failures and Repairs

Modelling failure of a truck is essential for model formulation. One can use several approaches to modelling failure in two dimensional warranties. Here, the one dimensional approach as inIskandar *et al.* [10] will be used and truck failures can be viewed as a one-dimensional point process. As mentioned before that a company operates a fleet of dump trucks to support its business. At the beginning of the lease period, the age of the product is A unit time. Given an item has survived for A time units and B usage units, the conditional failure rate of the used item after upgrading is given by $r_p(t|y) = r(A+t|y) - \rho_y\{r(A|y_1) - r(0|y_1)\},\ 0 < \rho_y < 1, A < t \le A+\tau$ where $y_1 = B/A$ is the used equipment's average usage rate during its past life; $0 < \rho_y < 1$ is the upgrade degree, which is used to describe the effect of upgrade action; while $r(0|y_1)$ is the initial failure intensity under usage rate y_1, which is non-zero. Note that p = 0 corresponds

to no reliability improvement; p = 1 indicates that the item is restored to as good as new; while p∈(0, 1) implies that the failure intensity is partially reduced. After upgrade, the used item's failure intensity at time t is influenced by the upgrade degree p, but its degradation pattern remains unchanged.
Let *Y* be the usage rate for a given truck. *Y* is considered varies across the trucks but it is constant for a given truck. For *Y* = *y*, the conditional hazard function is $r_y(t)$ which is a non-decreasing function of *t* (the age of the truck) and *y*. Usage rate of the truck and a land contour of a mining area where the truck is operated may strongly affect the degradation of the truck, that leads to failure.

One can use the accelerated failure time (AFT) model as in [10] to incorporate the effect of usage rate and the operating condition of the truck. Let y_0 denotes the nominal usage rate value associated with design reliability of the truck. Using the AFT formulation, if $T_0[T]$ is the time to first failure under usage rate $y_0[y]$ then we have $T_y = \left(y_0/y\right)^{\rho} T_0$ where ρ is a parameter representing the operating condition of a truck. A land contour of a mining site can be (i) a light incline, (ii) high incline or (iii) very hilly (i)-(iii) correspond to light, moderate, heavy operating conditions. We assign different value of ρ for different land contour of a mining site. The small, medium and large values of ρ will be assigned to represent light incline, high incline and very hilly, respectively. A larger value of ρ gives more stress to the truck and this in turn causes a larger effect on the truck's degradation.

We now model truck failures taking into account age, usage and operation condition. If $F_0(x;\alpha_0)$ is the distribution function for T_0 with scale parameter α_0 then the distribution function for T_y is the same as that for T_0 but with different scale parameter given by $\alpha_y = \left(y_0/y\right)^{\rho}\alpha_0$ where $\rho \geq 1$. Hence, we have $F(t,\alpha_y) = F_0((y_0/y)^{\rho}t,\alpha_y)$. The hazard and the cumulative hazard functions associated with $F(t,\alpha_y)$ are given by $r_y(t) = f(t,\alpha_y)/(1-F(t,\alpha_y))$ and $R_y(t) = \int_0^t r_y(x)dx$ respectively where $f(t,\alpha_y)$ is the associated density function. Since all failures are fixed by minimal repair and repair times are small relative to the mean time between failures, then failures occur according to a non-homogeneous Poisson process (NHPP) with intensity function $r_y\left(t\right)$.

E. Preventive Maintenance Policy.

The PM policy is defined as follows. For a truck with usage rate y, the PM policy is characterised by single parameter $\tau_y[\nu_y]$ during Ω_W [Ω_S]. Conditional on $Y = y$,the equipment is periodically maintained at $k\tau_y$, $k = 1, 2, \ldots$. This involves k disjoint intervals -$[0,\tau_y)$, …, $[l\nu_y,\Gamma_0)$ in which all failures within PM period are minimally repaired, where k is an integer value. Note $(k+1)\tau_y = W$ [$(\ell+1)\nu_y = W$] where $k[\ell]$ is an integer value. The effect of PM actions are modelled through the reduction in the intensity function after PM at t_j, $j \geq 1$ is δ. Since any failure occurring between PM is minimally repaired and $\delta_j^y = \delta$, then the expected total number of minimal repairs in $([t_{j-1},t_j), 1 \leq j \leq k+1)$ is given by $N = \sum_{j=1}^{k+1}\int_{t_{j-1}}^{t_j} r_{j-1}(t')dt'$.

2.1 Expected value for time to wait and repair and steady state distribution for X_{ji}

We consider a situation where a mining company operates a number of dump trucks (N) to fullfil a daily production target. Hence, there is a chance that more than one failed trucks are waiting to get repaired. Suppose that there are *k* failed trucks will be repaired by the OEM with a limited number of service channels (or servers), S. Here, we consider that a queue system of the OEM's service follows a Markovian queue with a finite population (N) and finite number of servers (S).

For truck $j(1 \leq j \leq N)$, if Z_j is the number of failures in $[0,\tau)$, T_{ji} is the time to failure after $(i-1)th$ repair $(2 \leq i \leq Z_j)$, $\tilde{T}_j$ is the time from the last repair to the end of the contract period, and X_{ji} $(1 \leq i \leq Z_j)$

is time needed to make the truck back to the operational state after the i-th failure (including waiting time and repair time), then we make the following assumptions:

1. Failed units are repaired on a first come first served basis.
2. Service contract period τ is sufficiently large in relation to mean time between failures so we can apply steady state condition for the distribution for X_{ji}.
3. The mean total waiting and repair times is very small in relation to the mean time tofailure or $E\left(X_{ji}\right) << E\left(T_{ji}\right) = 1/\lambda$ where λ is failure rate. As a result total down time for given y for each truck is small compare with $(W_{\bar{y}} + \tau)$, hence

$$K\left[\sum_{i=1}^{Z_j} T_{ji} + \tilde{T}_j - \sum_{i=1}^{Z_j} X_{ji}\right] \approx K\left[\sum_{i=1}^{Z_j} T_{ji} + \tilde{T}_j\right] \approx K(W_y + \tau); \mathrm{Z}_{\mathrm{j}}, 1 \le \mathrm{j} \le \mathrm{N}. \tag{1}$$

The arrival rate of failed truck and the departure (service) rate are given by

$$\lambda_k = \begin{cases} (N-k)\lambda & \text{for } 0 \le k \le N, \\ 0 & \text{for } k > N, \end{cases} \tag{2}$$

$$\mu_k = \begin{cases} k\mu & \text{for } 0 \le k \le S, \\ S\mu & \text{for } k > S. \end{cases} \tag{3}$$

if X_{ji} is the total (waiting and service) time in the system with its the density function $g(x)$ then,according toMurthy and Ashgarizadeh [1], the steady state density function for X_{ji} is

$$g(x) = e^{-\mu x}\sum_{k=0}^{S-1}\hat{P}_k + \sum_{k=0}^{N-1}\hat{P}_{k+s-1}\mu(S\mu)^k\left[\{e^{-\mu x}/(S\mu-\mu)^k\} - \sum_{k=0}^{k}[\{x^{k-j}e^{-S\mu x}\}/\{(k-j)!(S\mu-\mu)^k\}]\right], \tag{4}$$

where $\hat{P}_k$, P_k and P_0 k =1, 2, ..., N – 1 are given by

$$\hat{P}_k = \{(N-k)P_k\} \Big/ \left\{\sum_{k=0}^{N}(N-k)P_k\right\} \tag{5}$$

$$P_k = \begin{cases} (\lambda_y/\mu)^k\{N!/(N-k)!k!\}P_0 & \text{for } \quad k = 0,1,\dots,S-1 \\ (\lambda_y/S\mu)^k(S^S/S!)\{N!/(N-k)!k!\}P_0 & \text{for } \quad k = S, S+1,\dots,N \\ 0 & \text{for } \quad k > N \end{cases} \tag{6}$$

$$P_0 = \left[\sum_{S=0}^{S-1}(\lambda_y/\mu)^k\{N!/(N-k)!k!\} + \sum_{S=0}^{S-1}(\lambda_{\bar{y}}/S\mu)^k(S^S/S!)\{N!/(N-k)!k!\}\right]^{-1} \tag{7}$$

The expected value of X_{ji} for given y is

$$E_y[X_{ji}] = 1/\mu + \sum_{k=0}^{N-1}\frac{P_k\,(k-S+1)}{S\mu}. \tag{8}$$

A penalty occurs if the down time of a truck is above the target, $X_{ji} > \xi$ or the total down time caused by the i-th failure is greater than ξ . Hence, the probability that the penalty occurs at the i-th failure is given by $\mathrm{P}\left\{X_{ji} > \xi\right\}$. The expected penalty is given by

$$E_y\left[Max\left\{0, X_{ji} - \xi\right\}\right] = \int_{\xi}^{\infty}(x-\xi)g(x)dx = \int_{\xi}^{\infty}[1-G(x)]dx \tag{9}$$

where $G(x)$ is the distribution function of X_{ji} .The expression given by (9) is depent on P_k and λ_y, where λ_y is estimated by the mean value of failure intensity, $\bar{\lambda}_y$ (described in Section 3.2).

3.2 Expected value for number of failure times

Let $R_y(t)$ be the expected number of failures over $[0,t)$ for a given y if PM is outsourced to the OEM. Following the approach used in Jackson and Pascual [4], the mean value of failure intensity is approximated as $\bar{\lambda}_y = R_y^m(\mathrm{W}, \mathrm{W}_y + \tau)/(W_y + \tau)$ where $R_y^m(t) = \int_W^{W+\tau} r_y^m(x)dx$, $m = 1, 2$ referring to the expected number of failures for PM done in-house, and outsourced to the OEM, respectively.

1.1.1. Modelling Cost. The OEM's expected total cost consists of PM cost, repair cost, and penalty cost (incurred when the down time exceeds the predetermined target). If $J^1(k_y,\delta_y)$, $J^2(k_y,\delta_y)$ and $J^3(k_y,\delta_y)$ are the expected total PM cost, the expected total repair costs and the expected penalty cost over the MSC period $(W_y, W_y+L]$ for a given usage rate y, respectively, then the expected total cost for the OEM, $\Pi^n\left[k_y,\delta_y\right]$ is given by

$$\Pi^n\left[k,\delta_y\right] = J^1(k,\delta_y) + J^2(k,\delta_y) + J^3(k,\delta_y) \tag{10}$$

Let $C_{pm}(\delta_y)$ and C_r be the cost of the j-*th* PM and the cost of each minimal repair. If $C_{pm}(\delta_y) = C_0 + C_v\delta_y$ as in [5] and [6] then the expected total PM cost over the MSC period $(W_y, W_y+L]$ is given by

$$J^1(k_y,\delta_y) = \sum_{j=1}^{k} C_{pm}(\delta_y) = kC_0 + C_v\sum_{j=1}^{k}\delta_y \tag{11}$$

And the expected total minimal repair cost is given by

$$J^2(k,\tau_y,\delta_y) = C_r\left(R_y(L) - \sum_{j=1}^{k}\left(L - j\tau_y\right)\delta_y\right) \tag{12}$$

Let D and $\tilde{D}$ be the down time (consisting repair time and waiting time) for each failure and the down time allowed, respectively.The OEM incurs a penalty cost when $D > \tilde{D}$, and the penalty cost is assumed to be proportional to the excess of down time, $(D-\tilde{D})$. Then, the expected penalty cost is given by $J^3(k,\tau_y,\delta_y) = C_p\bar{G}(\mathrm{S})\left(R_y(L) - \sum_{j=1}^{k}\left(L - j\tau_y\right)\delta_y\right)$ where C_p is the penalty cost per unit time, $\bar{G}(\tilde{D})$= $\left\{\int_{\tilde{D}}^{\infty}\left(z-\tilde{D}\right)dF(z)\right\}$ is the expected value of penalty, and Z_i (downtime caused by the i-*th* failure) are assumed *i.i.d* with distribution function $F(z)$. After simplification we have the expected total cost of the OEM given by

$$\Pi^n\left[k,\tau_y,\delta_y\right] = \tilde{C}R_y(L) - \left\{\tilde{C}\sum_{j=1}^{k}\left[\left(L - C_v/\tilde{C}\right) - j\tau_y\right]\delta_y - kC_0\right\} \tag{13}$$

where $\tilde{C} = \left(C_r + C_p\bar{G}(\tilde{D})\right)$.

2.2.4. Modelling Profit . We assume that OEM and the owner have the same attitudes to risk, with the utility function , where ϕ is the owner's profit function in order to make the solution reach equilibrium.

Owner Expected Profit

The revenues for the owner consist of the revenue generated from the operation of the equipment plus the penalty cost paid by the OEM. Hence, the expected profit is given by

$$E\left[\phi_y\right] = K\left\{L - E[D_y^n(L)]\right\} + J^3(k_y,\delta_y) - P_G - C_b^n \tag{14}$$

where $E[D_y^n(L)] = \mu N_y$, μ (expected downtime), N_y (expected number of failures), K is the revenue ($/hour) received by the owner as a result of transporting mining materials from a mining area to a processing unit, and P_G is the MSC price.

OEM Expected Profit

The revenues for the OEM consist of the revenue received as a payment of the MSC contract plus the additional charges paid by the owner. The costs consist of PM cost, repair cost, penalty cost. Hence, the expected profit is given by

$$E\left[\pi_y\right] = P_G + \Phi(y) - \Pi^n\left[k_y, \delta_y\right] \tag{15}$$

The optimal PM interval for the OEM is obtained bymaximizing $E\left[\pi_y\right]$ with the respect to τ_y.

3 MODEL ANALYSIS

cost is $EC_{pm} = C_{pm}\tau$. As a result, the total expected profit of the OEM is

$$E\left[\pi(O_2)\right] = N\left\{\begin{matrix} P_G - C_{pm}\tau + R_y^2(\mathrm{W}, \mathrm{W}+\tau) \\ \left(C_I\int_0^{\xi}(\xi - x)\,g(x)\,dx - C_p\int_{\xi}^{\infty}(x-\xi)\,g(x)\,dx - C_m\right)\end{matrix}\right\} - C_0 S - C_1 S^2 \tag{16}$$

1.1.1. For $y > \gamma$, the expected profit of the OEM Modelling Cost. The OEM's expected total cost consists of PM cost, repair cost, and penalty cost (incurred when the down time exceeds the predetermined target). If $J^1(k_y,\delta_y)$, $J^2(k_y,\delta_y)$ and $J^3(k_y,\delta_y)$ are the expected total PM cost, the expected total repair costs and the expected penalty cost over the MSC period $(W_y, W_y+L]$ for a given usage rate y, respectively, then the expected total cost for the OEM, $\Pi^n\left[k_y,\delta_y\right]$ is given by

$$\Pi^n\left[k, \delta_y\right] = J^1(k,\delta_y) + J^2(k,\delta_y) + J^3(k,\delta_y) \tag{17}$$

Let $C_{pm}(\delta_y)$ and C_r be the cost of the j-*th* PM and the cost of each minimal repair. If $C_{pm}(\delta_y) = C_0 + C_v\delta_y$ as in [5] and [6] then the expected total PM cost over the MSC period $(W_y, W_y+L]$ is given by

$$J^1(k_y,\delta_y) = \sum_{j=1}^{k} C_{pm}(\delta_y) = kC_0 + C_v\sum_{j=1}^{k}\delta_y \tag{18}$$

And the expected total minimal repair cost is given by

$$J^2(k,\tau_y,\delta_y) = C_r\left(R_y(L) - \sum\nolimits_{j=1}^{k}\left(L - j\tau_y\right)\delta_y\right) \tag{19}$$

Let D and $\tilde{D}$ be the down time (consisting repair time and waiting time) for each failure and the down time allowed, respectively.The OEM incurs a penalty cost when $D > \tilde{D}$, and the penalty cost is assumed to be proportional to the excess of down time, $(D - \tilde{D})$. Then, the expected penalty cost is given by $J^3(k,\tau_y,\delta_y) = C_p\bar{G}(\mathrm{S})\left(R_y(L) - \sum\nolimits_{j=1}^{k}\left(L - j\tau_y\right)\delta_y\right)$ where C_p is the penalty cost per unit time, $\bar{G}(\tilde{D})$= $\left\{\int_{\tilde{D}}^{\infty}\left(z - \tilde{D}\right)dF(z)\right\}$ is the expected value of penalty, and Z_i (downtime caused by the i-*th* failure) are assumed *i.i.d* with distribution function $F(z)$. After simplification we have the expected total cost of the OEM given by

$$\Pi^n\left[k,\tau_y,\delta_y\right]=\tilde{C}R_y\left(L\right)-\left\{\tilde{C}\sum_{j=1}^{k}\left[\left(L-C_v/\tilde{C}\right)-j\tau_y\right]\delta_y-kC_0\right\} \tag{20}$$

where $\tilde{C}=\left(C_r+C_p\bar{G}(\tilde{D})\right)$.

2.2.4. Modelling Profit . We assume that OEM and the owner have the same attitudes to risk, with the utility function , where ϕ is the owner's profit function in order to make the solution reach equilibrium.

Owner Expected Profit

The revenues for the owner consist of the revenue generated from the operation of the equipment plus the penalty cost paid by the OEM. Hence, the expected profit is given by

$$E\left[\phi_y\right]=K\left\{L-E[D_y^n(L)]\right\}+J^3(k_y,\delta_y)-P_G-C_b^n \tag{21}$$

where $E[D_y^n(L)]=\mu N_y$, μ (expected downtime), N_y (expected number of failures), K is the revenue ($/hour) received by the owner as a result of transporting mining materials from a mining area to a processing unit, and P_G is the MSC price.

OEM Expected Profit

The revenues for the OEM consist of the revenue received as a payment of the MSC contract plus the additional charges paid by the owner. The costs consist of PM cost, repair cost, penalty cost. Hence, the expected profit is given by

$$E\left[\pi_y\right]=P_G+\Phi(y)-\Pi^n\left[k_y,\delta_y\right] \tag{22}$$

The optimal PM interval for the OEM is obtained bymaximizing $E\left[\pi_y\right]$ with the respect to τ_y.

4. OPTIMAL OPTION

1.1.1. Modelling Cost. The OEM's expected total cost consists of PM cost, repair cost, and penalty cost (incurred when the down time exceeds the predetermined target). If $J^1(k_y,\delta_y)$, $J^2(k_y,\delta_y)$ and $J^3(k_y,\delta_y)$ are the expected total PM cost, the expected total repair costs and the expected penalty cost over the MSC period $(W_y,W_y+L]$ for a given usage rate y, respectively, then the expected total cost for the OEM, $\Pi^n\left[k_y,\delta_y\right]$ is given by

$$\Pi^n\left[k,\delta_y\right]=J^1(k,\delta_y)+J^2(k,\delta_y)+J^3(k,\delta_y) \tag{23}$$

Let $C_{pm}(\delta_y)$ and C_r be the cost of the j-*th* PM and the cost of each minimal repair. If $C_{pm}(\delta_y)=C_0+C_v\delta_y$ as in [5] and [6] then the expected total PM cost over the MSC period $(W_y,W_y+L]$ is given by

$$J^1(k_y,\delta_y)=\sum_{j=1}^{k}C_{pm}(\delta_y)=kC_0+C_v\sum_{j=1}^{k}\delta_y \tag{24}$$

And the expected total minimal repair cost is given by

$$J^2(k,\tau_y,\delta_y)=C_r\left(R_y(L)-\sum_{j=1}^{k}\left(L-j\tau_y\right)\delta_y\right) \tag{25}$$

Let D and $\tilde{D}$ be the down time (consisting repair time and waiting time) for each failure and the down time allowed, respectively.The OEM incurs a penalty cost when $D > \tilde{D}$, and the penalty cost is assumed to be proportional to the excess of down time, $(D - \tilde{D})$. Then, the expected penalty cost is given by $J^3(k, \tau_y, \delta_y) = C_p \bar{G}(\mathrm{S})\left(R_y(L) - \sum_{j=1}^{k}\left(L - j\tau_y\right)\delta_y\right)$ where C_p is the penalty cost per unit time, $\bar{G}(\tilde{D})$= $\left\{\int_{\tilde{D}}^{\infty}\left(z - \tilde{D}\right) dF(z)\right\}$ is the expected value of penalty, and Z_i (downtime caused by the i-*th* failure) are assumed *i.i.d* with distribution function $F(z)$. After simplification we have the expected total cost of the OEM given by

$$\Pi^n\left[k, \tau_y, \delta_y\right] = \tilde{C}R_y\left(L\right) - \left\{\tilde{C}\sum_{j=1}^{k}\left[\left(L - C_v / \tilde{C}\right) - j\tau_y\right]\delta_y - kC_0\right\} \tag{26}$$

where $\tilde{C} = \left(C_r + C_p \bar{G}(\tilde{D})\right)$.

2.2.4. Modelling Profit . We assume that OEM and the owner have the same attitudes to risk, with the utility function , where ϕ is the owner's profit function in order to make the solution reach equilibrium.

Owner Expected Profit

The revenues for the owner consist of the revenue generated from the operation of the equipment plus the penalty cost paid by the OEM. Hence, the expected profit is given by

$$E\left[\phi_y\right] = K\left\{L - E[D_y^n(L)]\right\} + J^3(k_y, \delta_y) - P_G - C_b^n \tag{27}$$

where $E[D_y^n(L)] = \mu N_y$, μ (expected downtime), N_y (expected number of failures), K is the revenue ($/hour) received by the owner as a result of transporting mining materials from a mining area to a processing unit, and P_G is the MSC price.

OEM Expected Profit

The revenues for the OEM consist of the revenue received as a payment of the MSC contract plus the additional charges paid by the owner. The costs consist of PM cost, repair cost, penalty cost. Hence, the expected profit is given by

$$E\left[\pi_y\right] = P_G + \Phi(y) - \Pi^n\left[k_y, \delta_y\right] \tag{28}$$

The optimal PM interval for the OEM is obtained bymaximizing $E\left[\pi_y\right]$ with the respect to τ_y.

5 NUMERICAL EXAMPLE

Let us recall that for a given usage rate *y*, $F(t; y)$ is the time to the first failure which follows the Weibull distribution with $F_y(t; \alpha_y) = 1 - \exp(-t / \alpha_y)^{\beta}$. Its failure rate function is $r_y(t) = \beta\left(t^{\beta-1}/(\alpha_y)^{\beta}\right)$ where α_y as in (1). The other parameter values are:α_0 = 0.4, β = 2.5, W = 12 (months), L= 12 (months), U=24 (1x10^4Km), K= 24 (1x10^4Km) ($\gamma = U/W$ = 1), y_0 = 1, ρ = 1.5 and $C_v = 0.5C_m$, ζ =80 (hours) or 4 (days) .

Table 1 shows optimal solutions for cases 1 and 2. In Case 1, after warranty ends the optimization is carried out jointly for both players. As the usage rate increases, profit obtained for the agent (OEM) increases whilst the profit for the owner decreases.

Table 1. Results for Case 1 and Case 2

Warranty Region (W=12 months)		MSC Region (L=12 months)							
		Coordination				Non-Coordination			
$\bar{y}$	$k_y^*;\ \tau_y^*;\ E[\text{cost}]$	$\bar{y}$	$k_y^*;\ \tau_y^*;\ \delta_y^*$	$E[\pi_y];\ E[\phi_y]$ $(\times 10^5)$	$E[\Pi_y]$ $(\times 10^6)$	$k_y^*;\ \tau_y^*;\ \delta_y^*$	$E[\pi_y]=E[\phi_y]$ $(\times 10^5)$	P_G $(\times 10^5)$
1.2	6; 1.36; 866.07	1.2	5; 2.19; 1.09	2.04; 9.21	1.13	5; 2.18; 1.09	5.91	6.07
		2.0		2.20; 9.05			5.85	5.96
		3.0		2.30; 8.96			5.81	5.87
		4.0		2.38; 8.88			5.77	5.78

This is as expected since the agent performs a more effective PM (with $\boldsymbol{\delta}^*$ (= 1.09)) and the PM in turn will decrease the number of failures and hence the total maintenance cost and downtime. Compare with the profit resulting from the Nash bargaining solution (Case 2), the profit for the agent (OEM) and for the owner always equal. This is due to the bargaining strategy that maximize both profits. Since the number of failures increase as the usage increases then it in turn decreases the profit for both players. The similar result exists for the MSC prices, it decreases as the usage rate increases.

Tabel 2 shows the results (expected profit for each option, optimal option, and *S*(N)*) for ρ =1.2, 1.6, and 2.0 coresponding to light incline, high incline and very hilly, respectively. For a given usage rate (y=1.6) and land contour, *S*(N)* increases with N. This is to be expected since the set up cost for carrying out maintenance increases with S as the number of trucks increases. Note that if S >*S*(N)*, then the OEM 's expected profit decreases due to increase in the set up cost and operational cost.

For the OEM's and the owner, the maximum expected profit for Option 1 and Option 2 decrease with the increasing in land contour ρ . This is due to the equipment's availability decreases as the land contour influences the degradation of the truck (which increase the number of failures). Also shown in Table II, the optimal option for the OEM's and the owner are Option 2.

TABLE II: OPTIMAL OPTION FOR $\bar{y} = 1.6$ (*High usage rate*)

	ρ = 1.2					ρ = 1.6					ρ = 2.0				
	N					N					N				
	10	30	50	70	90	10	30	50	70	90	10	30	50	70	90
$E[O_1^*]$	12491(3)*	33666 (5)	47583 (7)	51525 (9)	43221 (10)	11295 (3)	27414 (6)	30550(9)	14571 (12)	No Profit	9434 (4)	17314 (8)	1816 (12)	No Profit	No Profit
C_S^*	13.58	14.51	15.92	17.81	17.67	8.15	9.12	10.89	13.36	-	4.99	6.10	8.05	-	-
$E[O_2^*]$	**12907 (2)***	**35822 (4)**	**53170 (6)**	**62046 (8)**	**64528 (9)**	**11911 (3)**	**30118 (5)**	**39107 (8)**	**33961 (10)**	**8996 (13)**	**10270 (4)**	**22215 (7)**	**9661 (12)**	No Profit	No Profit
P_G^*	**2304**	**1598**	**1403**	**1525**	**5.10[10]**	**1494**	**1211**	**1239**	**1172**	**5196**	**1329**	**1105**	**1168**	-	-
O^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	O_2^*	None	None
$S^*(N)$	**2**	**4**	**6**	**8**	**9**	**3**	**5**	**8**	**10**	**13**	**4**	**9**	**14**	-	-

*Number of optimal server in each option

6 CONCLUSION

In this paper we study a usage based MSC for dump trucks after the expired of a two-dimensional warranty, where the MSC is characterized by two parameters – age and usage limits which form a region. An imperfect PM (which reduces the age of the equipment) is performed during the MSC and the optimal imperfect PM is obtained by maximizing the expected total profit for the both players. The paper models the contract using the cooperative and non-cooperative game approach with one dimensional approach. One can models using a bivariate approach with multi players, and considers other shapes of a contract region. This is one topic for future research.

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REFERENCES

[1] Murthy, D. N. P. andAshgarizadeh E. 1999. Optimal decision making in a maintenance service operation, *European Journal of Operational Research*, 31, pp. 259–273.

[2] Ashgarizadeh, E. and Murthy, D. N. P. 2000. Service contracts, *Mathematical and Computer Modelling*, 31, pp. 11–20.

[3] Rinsaka, K. and Sandoh, H. 2006. A stochastic model on an additional warranty service contract, *Computers and Mathematics with Applications*, 51, pp. 179–188.

[4]Jackson, C and Pascual, R. 2008. Optimal maintenance service contract negotiation with aging equipment, *European Journal of Operational Research*, 189, pp. 387–398.

[5]Wang, 2010. A model for maintenance service contract design, negotiation and optimization, *European Journal of Operational Research*, 201(1), pp. 239 – 246.

[6] Wu, S. 2012. Assessing maintenance contracts when preventive maintenance is outsourced. *Reliabiliy Engineering and System Safety*, 98(1), 66 – 72.

[7] Iskandar, B.P., Pasaribu, U.S. and Husniah, H. 2013. Performanced Based Maintenance Contracts For Equipment Sold With Two Dimensional Warranties. Proc. of CIE43, Hongkong , pp.176–183,.

[8] Iskandar, B.P., Husniah, H. and Pasaribu, U.S. 2014. Maintenance Service Contract for Equipments Sold with Two Dimensional Warranties. *Journal of Quantitative and Qualitative Management* , (accepted).

[9] Pascual, R., Martinez, A., and Giesen, R. 2013. Joint optimization of fleet size and maintenance in a fork-join cyclical
transportation systems, *Journal of Operational Research Society*, 64, pp.982-994.

[10]Mirzahosseinian, H., and Piplani, R. 2011. Compensation and incentive modeling inperformance-based contracts for after market service. Proc. of the 41st Int. Conf. on Computing Industrial Engineering, Singapore, pp.739–744.

[11]Iskandar, B.P., Murthy, D.N.P. and Jack, N. 2005. A new repair-replace strategy for items sold with a two dimensional warranty.*Comp.andOper. Research*, 32(3),669–628.

[12]Osborne, M.J. and Rubinstein, A. 1994. *A Course in game theory*, Masssachusetts Institute of Technology.