PAPER • OPEN ACCESS

Fuzzy usage pattern in customizing public transport fleet and its maintenance options

To cite this article: H Husniah *et al* 2018 *J. Phys.: Conf. Ser.* **1013** 012186

IOP Conf. Series: Journal of Physics: Conf. Series **1013** (2018) 012186 doi:10.1088/1742-6596/1013/1/012186

Fuzzy usage pattern in customizing public transport fleet and its maintenance options

H Husniah[1]*, L Herdiani[1], Kusmaya[2] and A K Supriatna[3]

[1]Department of Industrial Engineering, Langlangbuana University, Bandung, West Java Indonesia
[2]Department of Informatics Engineering, Langlangbuana University, Bandung, West Java Indonesia
[3]Department of Mathematics, Padjadjaran University, Jatinangor, Sumedang, West Java Indonesia

*Corresponding author's e-mail address: hennie.husniah@gmail.com

Abstract. In this paper we study a two-dimensional maintenance contract for a fleet of public transport, such as buses, shuttle etc. The buses are sold with a two-dimensional warranty. The warranty and the maintenance contract are characterized by two parameters – age and usage – which define a two-dimensional region. However, we use one dimensional approach to model these age and usage of the buses. The under-laying maintenance service contracts is the one which offers policy limit cost to protect a service provider (an agent) from over claim and to pursue the owner to do maintenance under specified cost in house. This in turn gives benefit for both the owner of the buses and the agent of service contract. The decision problem for an agent is to determine the optimal price for each option offered, and for the owner is to select the best contract option. We use a Nash game theory formulation in order to obtain a win-win solution – i.e. the optimal price for the agent and the optimal option for the owner. We further assume that there will be three different usage pattern of the buses, i.e. low, medium, and high pattern of the usage rate. In many situations it is often that we face a blur boundary between the adjacent patterns. In this paper we look for the optimal price for the agent and the optimal option for the owner, which minimizes the expected total cost while considering the fuzziness of the usage rate pattern.

1. Introduction

Outsourcing becomes prevalent in many areas of industry, including the maintenance of vital equipment. This is more prevalent when the equipment is shopisticated and need a special expertise to maintain, such as modern buses in a public transport service system. In many public transport system, considering a complex and expensive equipments, an economical way to carry out maintenance is to outsource the maintenance works to an external agent. The agent can do a partial or full coverage of the maintenance actions, such as Preventive Maintenance (PM) or/and Corrective maintenance (CM). To meet the required maintenance actions, the owner wants to choose the best option which ensures that the equipment availability target is achieved with reasonable maintenance cost or maximum profit. On the other hand, the agent's problem is to determining the price of each option that maximises its profit.

IOP Conf. Series: Journal of Physics: Conf. Series **1013** (2018) 012186 doi:10.1088/1742-6596/1013/1/012186

Maintenance service contracts involving an incentives to motivate the the agent to increase the equipment's performance beyond the target have received a growing attention in the literature (see [1–6]). In contrast to all works previously, the authors in [7] studied a service contract which considers reduction cost to shield an agent against over claim and persue the owner to do PM and CM under specified cost in house, but it only deals with a single parameter – i.e. either age or usage. Furthermore the authors in [8] extended this single parameter to two dimensional service contracts and discussed a contract for the case where the owner has a fleet consists of N units of dump trucks sold with two- dimensional warranties. Two-dimesional approach has been used in many warranty papers which has been pioneered by [8,9].

As pointed by the authors in [10], two-dimensional approach in warranties has many advantages, such as the increase of competitive advantage and customer loyalty [11–15]. To increase the realism of warranty model, some complexities can easily incorporated into two-dimesional approach, such as the types of customer. As an example, the authors in [16] developed a two-dimensional warranty policy by considering two types of customers, i.e., customers with the adoption of policies concerning the warranty time or usage limit. Other type of customers can be characterized by the pattern of their usage rate [10]. To date, service contract after the expiry of a two-dimensional warranty which considering the types of customer usage rate is still limited. Only few literatures discussing this issue despite it is an important factor, since different types of customer give different optimal cost of maintenance contract [17].

The authors in [17] devised a service contract model and discussed a contract for the case where the owner has a fleet consists of N units of dump trucks sold with two- dimensional warranties. They consider two types of customer usage rate, i.e. medium and high usage rates. They used a crisp set membership of the usage rate level. In their hypothetical example, the usage rate of 1.35 is regarded as a medium usage rate while the usage rate of 1.75 is regarded as a high usage rate. It is not clear how to catagorized the usage rate in between those values, e.g. is 1.55 belong to medium or high usage rate? In reality, it is difficult to decide a membership of a value which lies just around a boundary. In this paper we extend the model in [17] to consider the vagueness of membership using the fuzzy set theory.The use of fuzzy set theory (including fuzzy numbers, fuzzy logic, fuzzy knowledge, and fuzzy decision rules) is not new in maintenance modeling. Even the authors in [18] emphasized that maintenance is among the areas where fuzzy sets have been applied intensively. Some of the literatures are highlighted in the following.

The authors in [19] gave an example of the application of fuzzy logic in maintenance decission making by considering the downtime of machines and the frequency of faults. The authors in [20] used a fuzzy knowledge based method for maintenance planning and applied it in a power system. Fuzzy logic is used to obtain adaptive preventive maintenance [21], scheduling preventive maintenance [22], imperfect maintenance [23], and recently it is used in [24] to model the imperfections of maintenance actions due to operators, in which they argued that the level of worker's skill is not crisp, so that the problem is well suited by the fuzzy set theoretical approach. The authors in [25] used fuzzy rules to interpret linguistic variables for determination of maintenance priorities, in which verbal expressions are among important factors in determining the priorities. Using this approach, the verbal expressions are quantified and used in decision making, which otherwise cannot be explicitly analyzed. To sum up, the authors in [26] argued that in reality there are always uncertainty of costs and reliability parameters in maintenance problems. This will rise problem if it is omitted by the model. They developed a maintenance model to accomodate the uncertainty by representing these parameters as fuzzy numbers and applied it to the wind turbine pitch control device. Their model would facilitate managers to make decisions based on a richer set of information.

In this paper we extend the model in [17] to consider the vagueness of membership boundary of customer usage rate using the fuzzy set theory. We look for the optimal price for the agent and the optimal option for the owner, in which the usage rate is known to be varying according to low, medium, and high level.

2. Methods

2.1. Model derivation

We model maintenance of a fleet of buses owned by a company. This problem is motivated by the work we carried out in a government public transport service Indonesia, known as DAMRI. As the owner of the fleet, so far the DAMRI does the maintenance by its own department, but for the future development it is planned to outsource the maintenance to the original equipment manufacturer (OEM) or the agent of the buses. The case which we model is the one in which neither owner nor OEM party is more powerful (symmetrical case). As a result, both parties will negotiate and determine jointly the terms and conditions of the service contract. To find the optimal solution we use Nash solution of the bargaining game of alternating offers for details [27,28].

The model presented here is exactly the same as the model in [17] with the addition of vagueness in usage rates. We assume that there will be three different type of usage rates, i.e. low, medium, and high usage rate. We proceed as follows, first we consider a crisp model and secondly we add the vagueness assumption in the numerical part as the input to the model. We will assume that the crisp function in the model propagates the fuzziness of independent variable (numerical input) to the dependent variable (numerical output). Since the model has been presented in [17], we will not discuss the detail derivation. All related concepts, such as warranty policy, maintenance service contract, and equipment failures and repairs are refered to [17]. We suggest the reader to consult [17] for the details of the notation and symbols used in the subsequent sections.

2.2. Model analysis

2.2.1. Owner decision problem. Suppose that there are k (k>1) failed buses will be served by the OEM with a single service channel using the first come, first served rule. The queue formed is considered as a Markovian. Then, the arrival rate of failed equipment is $\lambda_k = (N-k)\lambda$ for $0 \le k \le N$, where N is the number of bus population and λ is the arrival rate (or failure rate of a bus). The maintenance service rate is μ. According to [2], the steady state density function for z_j (waiting time for bus$j$) is

$$g(z) = \sum_{k=0}^{N-1} P_k \mu e^{-\mu y} (\mu z)^k / k!;\ k = 1,\ldots,N-1, \text{ where } P_k = \frac{(N-k)(\lambda/\mu)^k (N!/(N-k)!)}{\sum_{k=0}^{N-1}(N-k)(\lambda/\mu)^k (N!/(N-k)!)}, \quad (1)$$

The expression (1) depends on P_k and λ, where λ is estimated by the mean value of failure intensity, $\bar{\lambda}$ (see [2]). Let C denotes the repair cost to fix the failed equipment. We obtain the owner's expected profit for two options and each needs to consider two cases –i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

Option O_1: Case $y \le \gamma$, the expected profit is given by

$$\begin{aligned} E\left[\phi_y(O_1;C_s)\right] &= NK\left(\tau - E[D_y(W, W+\tau)]\right) \\ &\quad - N\left(C_s R_{1y}\left(W,(W+\tau)\right) + P_0 + C_b\right) \end{aligned} \quad (2)$$

where $R_{1y}\left(W, W+\tau\right) = \int_W^{W+\tau} r_{1y}(t)dt$ is expected number of failures. K is the revenue (Currency/hour) received by the owner as a result of transporting passengers from one station to another station. C_b and P_0 are the product cost and PM cost over the contract period, respectively. Sometimes, the failure of a bus can cause an accident. We consider that the probability of failure induces an accident is p, and hence the probability of failure does not induce accident is (1-p). We consider two types of downtimes, namely downtime 1 after a failure, downtime 2 after an accident caused by a failure. The downtime 2 of the bus is the sum of repair time and lost time due accident whilst downtime 1 is only a repair time. Down times 1 and 2 are exponentially distributed with parameter λ_1 and λ_2 respectively. As a result, $E[D_{1y}(W, W+\tau)] = R_{1y}\left(W, W+\tau\right)\left[(1-p)/\lambda_1 + p/\lambda_2\right]$. For case $y > \gamma$, the expected profit of the owner is given by (2) with the replacement of W by W_y.

IOP Conf. Series: Journal of Physics: Conf. Series **1013** (2018) 012186 doi:10.1088/1742-6596/1013/1/012186

Option O_2 :Case $y \le \gamma$, under Option 1, if repair cost, c is greater than ϑ then the owner has to pay $(c-\vartheta)$. As in many cases, the repair cost varies, then c is considered as a random variable with distribution function *G(c)*. As a result the expected additional repair cost paid by the owner is given by $\int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc$. The expected profit of the owner is given in (3), here $EP(\tau)$ is the expected penalty viewed as a compensation received by the owner (see OEM's Decision Problem). For $y > \gamma$,the expected profit of the owner choosing the option O_1 is similar in (3) with the replacement of W by W_y.

$$\begin{aligned} E\left[\phi_y\left(O_2;P_G\right)\right] &= N\left(K\left(\tau - E[D_{2y}(W,W+\tau)]\right) + EP(\tau)\right) \\ &- N(E\left[\text{Incentive cost}\right] + R_1(W,L)\int_{\vartheta}^{\infty}(c-\vartheta)g(c)dc \\ &+ P_G + C_b) \end{aligned} \tag{3}$$

2.2.2. OEM decision problem. Two cases need to be considered–i.e. (i) $y \le \gamma$ and (ii) $y > \gamma$.

Option O_1 **:** Case $y \le \gamma$, here with on call repair option, two costs incurs OEM i.e. repair cost and accident cost due to accident induced by failure. The expected profit of OEM for N units is given by

$$E\left[\pi_y\left(O_1;C_s\right)\right] = NR_y^1\left(W,W+\tau\right)\left(C_s - \left(C_m + C_a p\right)\right) \tag{4}$$

For case $y > \gamma$, the expected revenue of OEM is given by (4) with the replacement of W by W_y.

Option O_2 :Case $y \le \gamma$, the expected profit is given by

$$\begin{aligned} &E\left[\pi_y\left(O_2;P_G\right)\right] = P_G + E\left[\text{Incentive earned}\right] \\ &-E\left[\text{Repair and Compensation cost}\right] - E\left[\text{ Penalty cost}\right] \\ &-E\left[\text{PM cost}\right]. \end{aligned}$$

The expected of penalty cost $EP(\tau) = C_p\int_{\zeta}^{\infty}(z-\zeta)g(z)dz$, where ζ denotes the down time target of the equipment in (*0,t*) and the expected incentive cost is $EI(\tau) = C_I\int_0^{\zeta}G(z)\,dz$.

Let C_m and C_a be repair cost and compensation cost, respectively. The owner is required to pay an additional cost if the repair cost is greater than a threshold value ϑ. Then, the expected repair and conpensation costs is given by

$$EC(W,\tau) = (C_m + C_a p)R_{2y}\left(W,W+\tau\right) \tag{5}$$

where $C_m = \int_0^{\vartheta} cg(c)dc$ and $R_{2y}(W,W+\tau) = \int_W^{W+\tau} r_{2y}(t)dt$. The expected PM cost is $EC_{pm} = C_{pm}\tau$. As a result, we have the total expected profit of the OEM given by

$$E\left[\pi(O_2)\right] = N\left\{P_G - EP(\tau) + EI(\tau) - \mathrm{EC}(\tau) - C_{pm}\tau\right\} \tag{6}$$

For $y > \gamma$, the expected profit of the OEM choosing the option O_2 is given by (6) but it needs to replace W with W_y.

3. Results and Discussion

In this numerical example we vary the usage rate Y. For $Y=y$ the failure distribution is given by the Weibull distribution. Let the parameter values be as follows. α_0=1 (year), $1/\lambda = 1/300$ (year), $1/\lambda_2 = 1/(0.7\times 300)$, β =2.25, W=2 (years),U=2(3x10^{4}Km) ($\gamma = U/W = 1$), $y_0 = 1$, p_{acc}= 5% and m = 0.2, $\eta = 1.2$, $\tau = 1$ (years), υ=1(5x10^{4}Km). Other parameter values are K = 0.5(x10^3 \$), C_b = 16.87(x10^3 \$), C_{pm} = 0.003(x10^3 \$), C_m = 0.02(x10^3 \$), C_a = 3 C_m (x10^3 \$), C_p = 3K(x10^3 \$), C_i = 0.2K (x10^3 \$).

Assume that equipment operates 2,025 hours/year. The road condition in [17] is reflected by ρ = 1.2, 1.6 and 2.0 coresponding to light incline, high incline and very hilly, respectively. However, in our example we will only use one value of the road condition, i.e. ρ = 1.2.

To obtain the optimal number of fleet, we do the same technique as in [17]. Firstly, if we assume all of the input are crisp number, then we have the result as shown in Table 1. But now if we consider one of the input, i.e. the usage rate, is a fuzzy number rather than a crisp number, then the output certainly would be a fuzzy number. For example, the crisp number y = 1.55 in Table 1 now should be considered as a fuzzy number, let say it is represented as a triangular number $\bar{y}$ =(1.35,1.55,1.75). There are several ways to treat a fuzzy number as an input to a function. The author [29] pointed out that fuzzy function can be classified into three groups according to which aspect of the crisp function the fuzzy concept was applied, namely (i) Crisp function with fuzzy constraint, (ii) Crisp function which propagates the fuzziness of independent variable to dependent variable, and (iii) Function that is itself fuzzy. This fuzzifying function blurs the image of a crisp independent variable. In this paper we use the second approach, in which a crisp function propagates the fuzziness of independent variable to dependent variable.

Table 1. Output from crisp number of usage rate *y*

Y	*N**	*E[O₁*]*	*Cs**	*E[O₂*]*	*P_G**	*O**
0.95	62	NA	-	35022.83	18357.13	O_2
1.35	40	2689.93	6.93	14411.62	10546.63	O_2
1.55	34	4334.77	12.33	10719.10	7781.91	O_2
1.75	30	3189.91	9.742	8319.204	7640.92	O_2
1.95	27	2169.19	7.02	6650.19	7822.62	O_2
2.15	24	2877.00	10.01	5453.61	4997.71	O_2
2.75	20	NA	-	3227.27	9555.99	O_2

The computation of the formula gives the optimal option is O_2, i.e. to choose outsourcing by the OEM as the optimal maintenance strategy. By the use of the propagation of the fuzziness of independent variable to dependent variable we obtain the optimal fleet number N, the optimal price contract P_G, and the expected profit $E(O_2)$, respectively, given by:

$$\bar{N} = (30,34,40) = \begin{cases} 0 & ,N \le 30 \\ (N-30)/(34-30) & ,30 < N \le 34 \\ (N-40)/(34-40) & ,34 < N \le 40 \\ 0 & ,otherwise \end{cases}$$

$$\overline{P_G} = (7641,7782,10547) = \begin{cases} 0 & ,P_G \le 7641 \\ (P_G-7641)/(7782-7641) & ,7641 < P_G \le 7782 \\ (P_G-10547)/(7782-10547) & ,7782 < P_G \le 1054 \\ 0 & ,otherwise \end{cases}$$

$$\overline{E(O_2)} = (8319,10719,14412) = \begin{cases} 0 & ,E(O_2) \le 8319 \\ (E(O_2)-8319)/(10719-8319) & ,8319 < E(O_2) \le 10719 \\ (E(O_2)-14412)/(10719-14412) & ,10719 < E(O_2) \le 14412 \\ 0 & ,otherwise \end{cases}$$

Figure 1 gives the visual representation of these resulting fuzzy numbers. Due to the nonlinearity of the function involved, and the stopping criteria in the optimization process, the output is not necessarily a symmetrical triangular number or even is not necessarily a triangular number (e.g. see E[O_1*] in Table 1). It needs further investigation to reveal the shape of the resulting fuzzy number.

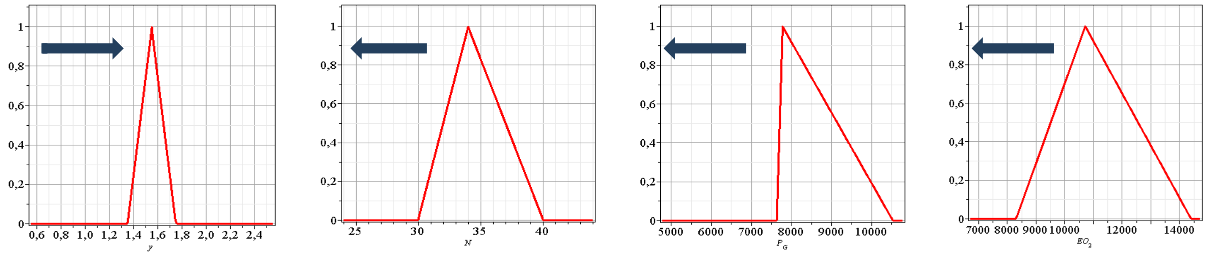

Figure 1. Membership function of the symmetrical triangular usage rate $\bar{y}$ =(1.35, 1.55, 1.75) as the input in the example (first figure) gives a nearly symmetrical triangular of the optimal fleet $\bar{N} = (30,34,40)$ and nearly symmetrical triangular of the optimal expected profit $\overline{E(O_2)} = (8319,10719,14412)$ (second and fourth figures), but produces a non-symmetrical optimal outsourcing price (third figure). The opposite direction of arrows indicate that the increase of the usage rate makes all the output decrease.

In the following discussion we assume that there are three different type of usage rates, i.e. low, medium, and high usage rate. The authors in [17] defined that the usage rate of 1.35 is regarded as a medium usage rate while the usage rate of 1.75 as a high usage rate. It is not clear how to catagorized the usage rate in between those values, e.g. is 1.55 belong to medium or high usage rate? We might assume the following fuzzy values for each usage category: (a) Triangular or Trapezoidal number and (b) Bell-shaped number, which give a symmetrical division of categories (see figures 2 and 3). They can also accomodate the blur boundary between the rate by the following fuzzy sets category (see [30] for details introduction on fuzzy sets and fuzzy numbers theory).

(a) Triangular and Trapezoidal Membership

$$\mu_{low}(y) = \begin{cases} 1 & , y \leq 0.95 \\ (y-1.35)/(0.95-1.35) & , 0.95 < y \leq 1.35 \\ 0 & , otherwise \end{cases}$$

$$\mu_{medium}(y) = \begin{cases} 0 & , y \leq 0.95 \\ (y-0.95)/(1.35-0.95) & , 0.95 < y \leq 1.35 \\ (y-1.75)/(1.35-1.75) & , 1.35 < y \leq 1.75 \\ 0 & , otherwise \end{cases}$$

$$\mu_{high}(y) = \begin{cases} 0 & , y \leq 1.35 \\ (y-1.35)/(1.75-1.35) & , 1.35 < y \leq 1.75 \\ 1 & , otherwise \end{cases}$$

(b) Bell-shaped Membership

$$\mu_{low}(y) = \begin{cases} exp(-((y-0.00)^\wedge 2/0.25)) & , -\delta \leq y \leq \delta \\ 0 & , otherwise \end{cases}$$

$$\mu_{medium}(y) = \begin{cases} exp(-((y-1.35)^\wedge 2/0.5)) & , -\delta \leq y \leq \delta \\ 0 & , otherwise \end{cases}$$

$$\mu_{high}(y) = \begin{cases} exp(-((y-2.75)^\wedge 2/0.25)) & , -\delta \leq y \leq \delta \\ 0 & , otherwise \end{cases}$$

The graph of the membership are shown in Figures 2 and 3.

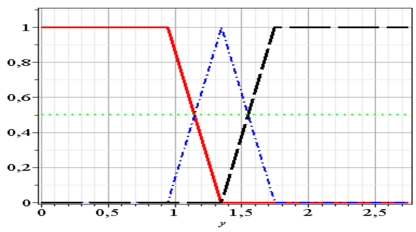

Figure 2. Membership functions of the fuzzy subsets with trapezoidal and triangular shape. Low (red line), Medium (blue dash-dot), and High (black long-dash) usage rates.

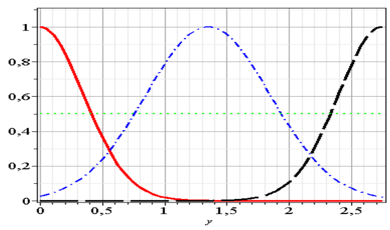

Figure 3. Membership functions of the fuzzy subsets with bell shape curve. Low (red line), Medium (blue dash-dot), and High (black long-dash) usage rates.

4. Conclusion

In this paper we give an example on how to accommodate the fuzziness of a parameter in determining the optimal fleet number and the optimal strategy for maintenance the fleet. In general, for a given crisp value y and a fixed reliability level, the optimal expected profit decreases as the usage rate y increases. This is become more prevalent in the presence of fuzziness, by observing the output of the low, medium, and high usage rates, which gives the relatively high, medium, and low numbers of fleet, profit, and price of the optimal option. In reality, the usage rate of a bus increases due to a longer travelled distance from a station to other station. This is as expected since the increasing in y causes the failure rate to increase and this in turn increases the number of failures under the maintenance contract. Hence the total revenues (and so that the profit) for both parties decrease. This in turn will decrease the price of the contract and the expected profit for each party as the total surplus generated needs to be shared. Different shape of membership can be explored to investigate the sensitivity of the results presented in this paper.

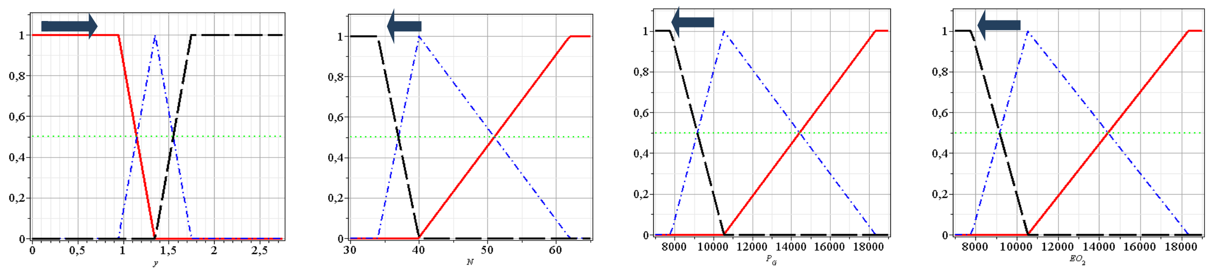

Figure 4. The first figure shows the membership functions of the fuzzy subsets with trapezoidal and triangular shape. There are three categories of the usage rates, i.e. low (red line), medium (blue dash-dot), and high (black long-dash) usage rates. All the membership of the output numbers (the fleet number, the price of the optimal maintenance option, and the expected revenue are conform).

5. References

[1] Rinsaka K and Sandoh H 2006 *Computers and Mathematics with Applications* **51** 179–188
[2] Jackson C and Pascual R 2008 *European Journal of Operational Research* **189** 387–398
[3] Iskandar B P, Husniah H and Pasaribu U S 2014 *Quality Technology and Quantitative Management* **11** 321–333
[4] Iskandar B P, Cakravastia A, Pasaribu U S and Husniah H 2014 *Technology, Informatics, Management, Engineering, and Environment (TIME-E), 2014 2nd International Conference* 117-122 IEEE
[5] Mirzahosseinian H and Piplani R 2011 Compensation and incentive modeling in performance-based contracts for after market service *Proc. Int. Conf. Computers and Industrial Eng.* 739-744
[6] Husniah H, Pasaribu U S, Cakravastia A and Iskandar B P 2014 Performance-based maintenance contract for equipments used in mining industry *Proc. of IEEE Int. Conf. Manag. of Innovation and Tech. (ICMIT)* 6942485
[7] Husniah H, Pasaribu U S and Iskandar B P 2015 *ARPN J. of Eng. and App. Sci.* **10** 146–151
[8] Iskandar B P, Murthy D N P and Jack N 2005 *Comp. and Oper. Research* **32** 669–628
[9] Iskandar B P and Murthy D N P 2003 *Math. and Comp. Modelling* **38** 1233-1241
[10] Huang Y S, Huang C D and Ho J W 2016 *E. J. of Op. Research* **257** 3 971-978
[11] Majeske K D 2007 *Reliability Eng. and Sys. Safety* **92** 2 243-251
[12] Huang Y S and Yen C 2009 *IIE Transactions* **41** 4 299-308
[13] Shafiee M, Chukova S, Saidi-Mehrabad M and Niaki S T A 2011 *Communications in Statistics-Theory and Methods* **40** 4 684-701
[14] Su C and Shen J 2012 *Eng. Failure Ana.* **25** 49-62
[15] Shahanaghi K, Noorossana R, Jalali-Naini S G and Heydari M 2013 *Eng. Failure Analysis* **28**

90-102
[16] Huang Y S, Gau W Y and Ho J W 2015 *Reliability Eng. & Sys. Safety* **134** 51-58
[17] Husniah H, Pasaribu U S, Supriatna A K and Iskandar B P 2017 Optimal number of fleet maintenance contract with policy limit cost *Control, Decision and Information Technologies CoDIT 2017 4th International Conference* 0876-0880 IEEE
[18] Strackeljan J and Weber R 1999 *Quality control and maintenance. In Practical applications of fuzzy technologies* 161-184 (Boston: Springer)
[19] Wolkenhauer O 2001 *Data Engineering: Fuzzy Mathematics in System Theory and Data Analysis* (John Wiley and Sons)
[20] Sergaki A and Kalaitzakis K 2002 *Reliability Eng. and Sys. Safety* **77** 19-30
[21] Yuniarto M N and Labib A W 2006 *Int. J. of Prod. Research* **44** 1
[22] Fouad R H and Samhouri M 2011 A Fuzzy Logic Approach for Scheduling Preventive Maintenance in ERP System *Management and Service Science (MASS) 2011 International Conference*1-4 IEEE
[23] Hennequin S, Arango G and Rezg N 2009 *J.of Quality in Maint. Eng.* **15** 4 412-429
[24] Segura M A, Hennequin S and Finel B 2016 *Int. J. of Op. Research* **27** 1-2
[25] Khanlari A, Mohammadi K and Sohrabi B 2008 *Computers & Industrial Engineering* **54** 2 169-184
[26] Carvalho M, Nunes E and Telhada J 2015 *Int. J. Prod. Manag. Eng* **3** 2 103-109
[27] Osborne M J and Rubinstein A 1994 *A Course in Game Theory* (Cambridge: Masssachusetts Institute of Technology Press)
[28] Binmore K, Rubinstein A and Wolinsky A 1986 *Journal of Economics* **17** 2 176-188
[29] Lee K H 2005 *First Course on Fuzzy Theory and Applications* (Verlag, Heidelberg: Springer)
[30] Barros L C, Bassanezi RC and Lodwick W E 2017 *A First Course in Fuzzy Logic, Fuzzy Dynamical Systems, and Biomathematics* (Verlag, Heidelberg: Springer)

Acknowledgments
The authors would like to thank the anonymous referee for the valuable comments which improved significantly the earlier version of the paper. This work is funded through the scheme of “Hibah PUPT 2017 SP DIPA-042.06.1.401516/2017” KEMENRISTEKDIKTI Indonesia.