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A mathematical model of predicting residual moment capacity of RC elements after fire exposure

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Abstract

Purpose – This paper aims to present a mathematical model of predicting the residual moment capacity of fire-damaged reinforced concrete (RC) elements after cooling to ambient temperature which also reflects the role of bond between steel rebar and surrounding concrete.

Design/methodology/approach – The prediction of residual moment capacity of fire-damaged RC element has been carried out for two scenarios: by assuming perfect bond between surrounding concrete and steel rebar after fire exposure and by incorporating a relative slip between surrounding concrete and steel rebar and hence assuming partial bond between them after fire scenario. The predicted results are then compared with the experimental results available in different literatures.

Findings – It is found that on comparison between the predicted results and the experimental results, the proposed mathematical prediction model, when bond-characteristics are considered, shows better agreement with the experimental results as compared with those by conventional method with perfect bond assumption.

Originality/value – The constitutive relationship for thermal residual properties of steel rebar and concrete has been used in the proposed prediction model along with relative slip approach between surrounding concrete and steel rebar after fire scenario and consequently to predict the residual moment capacity of the fire-damaged RC element after cooling.

Keywords Air cooling, Fire exposure, Loss of strength, Residual bond strength

Paper type Research paper

Notations

As	= Total area of tension reinforcement;
A's	= Area of compression reinforcement;
As1	= Part of the tension reinforcement in equilibrium with the concrete compression block;
As2	= Part of tension reinforcement in equilibrium with the compression reinforcement;
Ai	= Cross-section area of reinforcement bar i;
a	= Distance between bottom surface of the effective cross-section to the centroid of the reinforcement;
az	= Depth of damaged zone;
b	= Width of RC beam section at ambient temperature;
bfi	= Width of fire-reduced effective cross-section;
c	= Clear cover;
d	= Effective depth of RC beam section at ambient temperature;
dfi	= Effective depth of the fire-reduced effective cross-section;
D	= Overall depth of RC beam section at ambient temperature;

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Ec	=	Modulus of elasticity of concrete at ambient temperature;
EcT	=	Residual modulus of elasticity of concrete after heating to elevated temperature T;
Es	=	Modulus of elasticity of steel rebar at ambient temperature;
EsT	=	Residual elastic modulus of steel rebar after heating to elevated temperature T;
fc	=	Cylindrical compressive strength of concrete at ambient temperature;
fcT	=	Residual cylindrical compressive strength of concrete after heating to elevated temperature T;
f'cT	=	Cylindrical compressive strength of concrete at elevated temperature T;
fscy	=	Yield strength of compression reinforcement at ambient temperature;
fscyT	=	Residual yield strength of compression reinforcement after heating to elevated temperature T;
fy	=	Yield strength of tension reinforcement at ambient temperature;
fyT	=	Residual yield strength of tension reinforcement after heating to elevated temperature T;
FT	=	Flexural tensile force;
Fc	=	Total compressive force;
kcT	=	Residual strength reduction factor of concrete after heating to elevated temperature T;
kcTi	=	Residual strength-reduction factor of concrete after heating to elevated temperature T for ith concrete layer in beam section;
kc,M	=	Mean residual strength-reduction factor of concrete after heating to elevated temperature T;
kc(TM)	=	Reduction coefficient for concrete at mid-slice;
ksT	=	Residual strength reduction factor of steel rebar after heating to elevated temperature T;
kτbT	=	Residual bond strength reduction factor due to fire exposure;
l	=	Effective span of beam;
l0	=	Span length for constant moment zone or distance between two point loads;
n	=	Number of steel rebars;
T	=	Temperature in °C;
Tf	=	Furnace temperature in °C;
t	=	Fire exposure time in hours;
x (or xT)	=	Depth of neutral axis;
z	=	Lever arm between the tension reinforcement and the concrete;
z'	=	Lever arm between the tension reinforcement and the compression reinforcement;
ϕ	=	Diameter of steel rebar;
λ	=	Depth of stress block factor;
η	=	Effective strength factor;
ΔM	=	Difference between experimental and predicted residual moment capacity of a fire-damaged RC beam;
ΔT	=	Rise of temperature in steel or concrete material over ambient temperature;
εc	=	Strain in concrete at ambient temperature;
εcT	=	Strain in concrete after elevated temperature T;
εc1,T	=	Strain corresponding to peak stress after elevated temperature T;
εc1'T	=	Strain corresponding to peak stress at elevated temperature T;
εcuT	=	Ultimate compressive strain in the concrete after elevated temperature T;
εcu'T	=	Ultimate compressive strain in the concrete at elevated temperature T;
εs	=	Strain in steel rebar at ambient temperature;
εsT	=	Strain in steel rebar after elevated temperature T;
εy	=	Yield strain in steel rebar at ambient temperature;
εyT	=	Yield strain in steel rebar after elevated temperature T;
σcT	=	Stress in concrete after elevated temperature T;

1. Introduction

When fire disaster occurs in any reinforced concrete (RC) structure, then different parts of it may get exposed to different temperatures during or after fire exposure. It causes reduction in the strength and stiffness of both concrete and steel significantly and also affects other physical properties even after being cooled down to ambient temperature. The RC structure on exposure of fire may undergo complete or partial damage or even collapse. In case of such partially damaged concrete structures, it becomes important to have an assessment of their residual strength or structural capacity for carrying out the necessary retrofitting work to bring them back in service, if possible. To achieve this, it is important to have the knowledge of the residual properties of steel rebars and concrete and that of the bond between steel rebar and surrounding concrete at temperature experienced in fire (Hassan, 2012).

The prediction approach of residual (after cooling) moment capacity of RC elements after fire exposure has not been discussed in detail in the available literature. However, Kodur *et al.* (2010) proposed a relationship for a rough estimate of the residual moment capacity of RC elements exposed to fire. Hsu *et al.* (2006) developed a computer program to predict residual moment capacity of fire-damaged RC beams by dividing the entire beam into a number of segments for analysis based on the lumped system concept. There also, quite limited experimental investigations have been performed for determination of residual moment capacity of RC flexural elements after being exposed to fire (design/standard fire). The results of those experimental investigations showed that the residual moment capacity of RC beams gets decreased with an increase in temperature and also with the increase in exposure period to fire (Kodur *et al.*, 2010; Prasad *et al.*, 2010; Prasad *et al.*, 2012; Mahdi *et al.*, 2009; Kumar, 2003; Moetaz *et al.*, 1996). The different standard codes of different countries are also silent about the prediction of the residual (after cooling) moment capacity of fire-damaged RC flexural elements. However, Eurocode 2 (2005) presents some prediction approaches for residual moment capacity of RC elements subjected to elevated temperature only. The role of bond between steel rebar and surrounding concrete has never been brought into the light for predicting the residual (after cooling) moment capacity of RC flexural elements after fire exposure.

This paper presents a mathematical model of predicting the residual moment capacity of fire-damaged RC elements after cooling which also reflects the role of bond between steel rebar and surrounding concrete. The residual moment capacity has been predicted by assuming a perfect bond and a partial bond between steel rebar and surrounding concrete in fire-damaged RC element after cooling.

2. Constitutive relationships for material properties after fire exposure

2.1 Constitutive relationship for residual material properties of concrete

In this analysis, residual material properties for thermally affected concrete after cooling suggested by Eurocode 4 (Annex C: EN 1994-1-2:2005 and Figure 3.2: EN 1994-1-2:2005) Eurocode 4 (2005) has been adopted. This model of compressive behavior of the concrete material is represented as a compressive nonlinear stress–strain curve as a function of concrete temperature developed because of fire exposure. The linear stress–strain relationship in the softening zone has been permitted:

$$\frac{f_{cT}}{f_c} = k_{cT}; 20°C \leq T < 100°C \tag{1a}$$

$$= 1.0 - 0.235 * \frac{(T - 100)}{200}; 100°C \leq T < 300°C \tag{1b}$$

$$= 0.9 * k_{cT}; T \geq 300°C \tag{1c}$$

And:

$$\sigma_{cT} = f_{cT}\left[\frac{3 * \left(\frac{\varepsilon_{cT}}{\varepsilon_{c1T}}\right)}{2 + \left(\frac{\varepsilon_{cT}}{\varepsilon_{c1T}}\right)^3}\right]; 0 \leq \varepsilon_{cT} \leq \varepsilon_{c1,T} \tag{2}$$

For $\varepsilon_{c1,T} \leq \varepsilon_{cT} \leq \varepsilon_{cu,T}$, linear models are permitted:

$$\text{For } \sigma_{cT} = 0,\ \ \varepsilon_{cu,T} = \varepsilon_{c1,T} + \left[\frac{(\varepsilon_{cu',T} - \varepsilon_{c1',T}) * f_{cT}}{f_{c'T}}\right] \tag{3}$$

where:

$$\varepsilon_{cu',T} = 2 * 10^{-10}T^3 - 1 * 10^{-7}T^2 - 0.00003T + 0.0018; T < 600°C \tag{4a}$$

$$= 0.0250; T \geq 600°C \tag{4b}$$

$$\varepsilon_{c1',T} = 7 * 10^{-10}T^2 - 0.00003T + 0.0197;\ 20°C \leq T \leq 1{,}100°C \tag{5}$$

$$f_{c'T} = k_{cT} * f_c \tag{6}$$

$k_{cT} =$ strength reduction factor (Table 3.1: EN 1992-1-2:2004) (Eurocode 4, 2004a).

2.2 Constitutive relationship for residual material properties of steel rebar

In this study, Tao *et al.*'s (2013) model has been used for the simulation of residual material properties of thermally affected steel rebars after cooling. This model is developed to analyze the effects of heat exposure on key parameters, such as the residual modulus of elasticity, residual yield strength and residual ultimate strength, which control the full-range stress–strain curves of steel in post-fire scenario (Tao and Wang, 2013). Degradation of the material properties of the steel rebar due to fire exposure was incorporated in the model by using temperature-dependent reduction factors:

$$\frac{f_{yT}}{f_y} = 1; T \leq 500°C \tag{7a}$$

$$= 1 - 5.82 * 10^{-4}(T - 500); T > 500°C \tag{7b}$$

$$\frac{E_{sT}}{E_s} = 1; T \leq 500°C \tag{8a}$$

$$= 1 - 1.30 * 10^{-4}(T - 500); T > 500°C \tag{8b}$$

$$\varepsilon_{yT} = \frac{f_{yT}}{E_{sT}} \tag{9}$$

2.3 Constitutive relationship for residual bond strength between steel rebar and surrounding concrete

In this study, Chiang and Tsaib's (2009) model has been adopted for the simulation of residual bond strength between steel rebar and surrounding concrete in a thermally affected

RC element after cooling. This model gives the relationship for normalized residual bond strength in terms of fire exposure time. Through the regression analysis of Chiang and Tsaib's (2009) model of residual bond strength between concrete and reinforcing steel bar after fire exposure, the following relationships have been developed and adopted for residual bond strength in terms of exposure temperature as:

$$\frac{\tau_{bT}}{\tau_b} = 8.6124 * T^{-0.448}; 20°C \leq T \leq 1{,}000°C \tag{10}$$

In addition, following relationships consider the effect of fire duration on the bond between steel rebar and concrete as follows:

For 20°C ≤ T ≤ 1,100°C and t = 30 min:

$$\frac{\tau_{bT}}{\tau_b} = -7 * 10^{-10}T^2 - 0.0007T + 1.0185 \tag{11}$$

For 20°C ≤ T ≤ 1,000°C and t = 60 min:

$$\frac{\tau_{bT}}{\tau_b} = 5 * 10^{-8}T^2 - 0.001T + 0.9979 \tag{12}$$

For 20°C ≤ T ≤ 900°C and t = 90 min:

$$\frac{\tau_{bT}}{\tau_b} = -1 * 10^{-9}T^3 + 2 * 10^{-6}T^2 - 0.0018T + 1.0309 \tag{13}$$

For 20°C ≤ T ≤ 800°C and t = 120 min:

$$\frac{\tau_{bT}}{\tau_b} = -2 * 10^{-9}T^3 + 3 * 10^{-6}T^2 - 0.0023T + 1.0445 \tag{14}$$

3. Calculation of temperatures

In any RCC element, the temperatures, within a concrete element including therefore those of steel rebar, continue to rise for a period after the maximum fire temperature has been reached. Wickstrom(1986) has proposed a method of calculating that rise in temperature for biaxial heat flow:

$$\text{Rise in temperature } (\Delta T) = (n_w(n_x + n_y - 2n_xn_y) + n_xn_y)\Delta T_f \tag{15}$$

where:

$n_w = 1 - 0.0616t^{-0.88}$

$$n_x = 0.18ln\left(\frac{a}{a_c} * \frac{t}{x^2}\right) - 0.81 \tag{16}$$

where:

a = thermal diffusivity of the concrete under consideration in m^2/s;
a_c = reference value = 0.417×10^6 m^2/s.

For $a = a_c$:

$$n_x = 0.18\, ln\left(\frac{t}{x^2}\right) - 0.81 \tag{17}$$

with x (or y) subject to the limit:

$$x \geq 2D - 3.6\sqrt{(0.0015t)} \tag{18}$$

where:

x (or y) = depth into the member (in m).

4. Determination of effective area after fire exposure

When any RC structural element gets exposed to fire, there may be chance of occurrence of spalling. Because of this spalling effect, the sectional dimensions of the concrete beam get reduced depending upon the temperature attained in concrete layers. In any fire-damaged RCC beam, the reduced effective width (b_{fi}) and effective depth (d_{fi}) can be obtained by making use of 500°C isotherm curves or by method of slices. A general reduction of the cross-section size of the RC element is performed depending upon the temperature attained at the concrete surfaces.

For calculation of effective width (b_{fi}) and effective depth (d_{fi}), 500°C isotherm curves for different fire resistances can be used. The thickness of the damaged concrete is made equal to the average depth of the 500°C isotherm in the compression zone of the cross section (Eurocode 2, 2004a). Wickstrom (1986) has proposed a relationship for getting 500°C isotherm curves for position x for a temperature rise ΔT_x at time t and furnace temperature ΔT_f as follows:

$$x = \left[\frac{\dfrac{a}{0.417 * 10^{-6}} \times t}{\left(4.5 + \dfrac{\Delta Tx}{0.18 n_W \Delta T_f}\right)}\right]^{0.5} \tag{19}$$

For the 500°C isotherm, $\Delta T_x = 480$°C and $x = x_{500}$. For a given value of thermal diffusivity (a) of any aggregate type and exposure time (t), the damage zone x_{500} similar to 500°C isotherm curves can be generated (Figure 1).

The method of slices can also be used to determine fire-damaged effective width (b_{fi}) and effective depth (d_{fi}). In this method, the heat-affected concrete is divided into a series of slices the temperature (T) determined at the mid-depth of each slice. The mean concrete strength reduction factor $k_{c,m}$ is then calculated as:

$$k_{c,m} = \frac{1 - \dfrac{0.2}{n}}{n} \sum_{i=1}^{n} k_{cTi} \tag{20}$$

where:

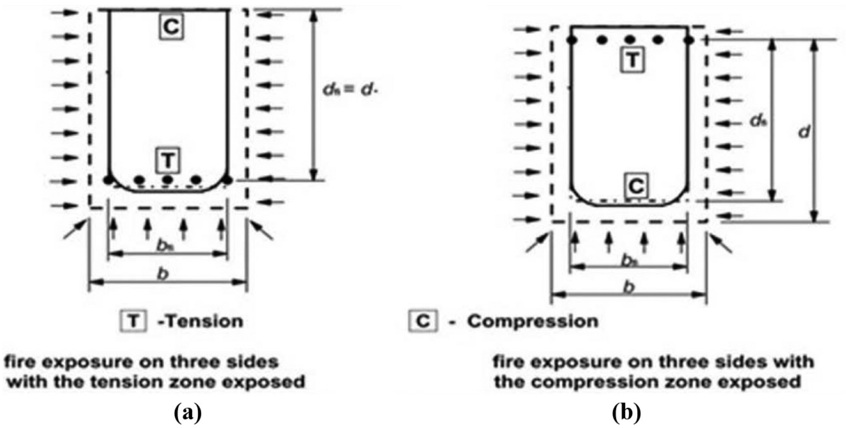


Figure 1.
Reduced cross section of reinforced concrete beam after fire exposure

n = number of slices.

k_{cTi} = concrete strength reduction factor at elevated temperature (T) for ith slice, and this can be taken from Eurocode 2 (Table 3.1: EN 1992-1-2:2004) (Eurocode 2, 2004a).

Then, the effective sectional dimensions can be calculated by reducing the width of the damage zone from original sectional dimensions (width, b, and depth, d):

$$\text{Width of damage zone, } a_z = \frac{b}{2}\left[1 - \frac{k_{c,m}}{k_c(T_M)}\right] \tag{21}$$

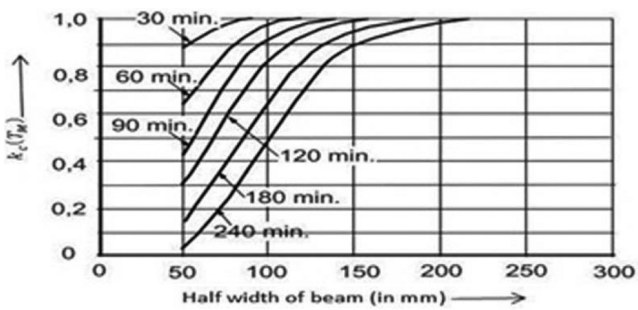
where:

$k_c(TM)$ = strength reduction factor (Figure 2).

5. Conventional prediction approach with perfect bond assumption

In this section, the residual moment capacity of fire-damaged RC element after cooling has been predicted in conventional manner as follows:

- This prediction approach uses the assumption of existence of a perfect bond between steel rebar and surrounding concrete in the RC element even after fire exposure.



Source: Euocode 2 (2004a)

Figure 2.
Strength reduction factor for a reduced cross section using siliceous aggregate concrete

- As per Eurocode 2 (Annex B: EN 1992-1-2) (Eurocode 2, 2004a), the residual moment capacity of fire-damaged RC beam after cooling can be calculated conventionally by making use of 500°C isotherm method and slice method.
- Similar prediction steps have also been adopted for modified 500°C isotherm method. This method is quite similar to 500°C isotherm method, but it uses mean residual compressive strength of concrete (f_{cT}) after fire exposure at temperature T in place of compressive strength of concrete at ambient temperature (f_c) for the calculation of residual moment capacity of fire-exposed RCC beam after cooling.

6. Slip approach with partial bond assumption

Although the flexural capacity calculation approach of fire-damaged RCC beam based on the perfect bond assumption between steel rebar and surrounding concrete uses the assumption that both the strain compatibility and equilibrium condition must be satisfied. However, if the beam is damaged because of fire exposure, bond degradation occurs, which cause a premature slip, which leads to an unknown bond–slip relationship which starts working there, and hence, the tensile strain in concrete will not be well transferred to reinforcement and hence the strain of steel may be different from that of concrete at the same level. Hence, a new strain compatibility relationship must be developed to enable a more realistic analysis of a fire-damaged RC elements.

The distribution of strain for a perfectly bonded RC beam, i.e. beam at ambient temperature as shown in Figure 3, is given as equation (22):

$$\frac{\varepsilon_S}{\varepsilon_C} = \frac{d - x}{x} \tag{22}$$

where:

d = effective depth of beam section;
x = depth of compression zone;
εs = stain in steel at ambient temperature;
εc = strain in concrete at ambient temperature.

For an RC beam unbonded along the whole span of the beam except for the anchorage length, Wang and Liu (2008) . have given the following strain compatibility relationship:

$$\frac{\varepsilon_S^{ub}}{\varepsilon_C^{ub}} = \frac{l_0}{l} * \left(\frac{d - x_{ub}}{x_{ub}}\right) \tag{23}$$

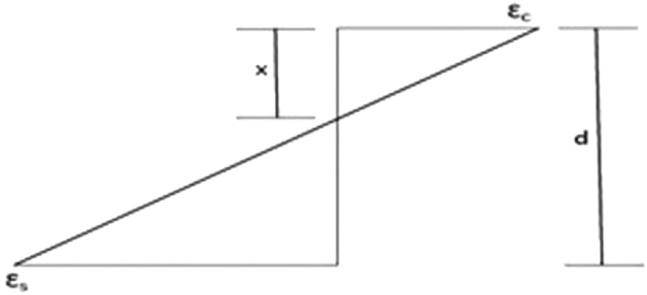


Figure 3.
Strain distribution for a perfectly bonded RC beam

where:

x_{ub} = depth of compression zone in unbonded beam;
ε_s^{ub} = stain in steel at ambient temperature in unbonded beam;
ε_c^{ub} = strain in concrete at ambient temperature in unbonded beam;
l_0 = span length for constant moment zone;
l = effective span length.

According to Wang and Liu (2008), the strain compatibility relationship for partially bonded beam can be assumed in between that of perfectly bonded and unbonded beam. Hence, in this study, noticing the form of equations (22)-(23) and assuming that the strain compatibility of the fire-damaged beam, i.e. partially bonded beam, lies in between that of perfectly bonded and unbonded beam. The strain compatibility of the fire-damaged beam, i.e. partially bonded beam, has been assumed as:

$$\frac{\varepsilon_{sT}}{\varepsilon_{cT}} = g(\tau_{bT}) \cdot \left(\frac{d - x_T}{x_T} \right) \tag{24}$$

where:

ε_{sT} = stain in steel after temperature T in fire-damaged beam (after cooling).
$= \sigma_{sT}/E_{sT}$
σ_{sT} = stress in steel rebar after temperature T in fire-damaged beam (after cooling).
ε_{cT} = stain in concrete after temperature T in fire-damaged beam (after cooling).
$g(\tau_{bT})$ = interpolation factor as function of temperature T.
$x = x_T$ = depth of neutral axis after temperature T in fire-damaged beam (after cooling).

Interpolation factor $g(\tau_T)$ for strain compatibility relationship for partially bonded RC element is obtained by making linear interpolation between unbonded and perfectly bonded condition which is given as:

$$g(\tau_{bT}) = 1 - \left[\left(1 - \frac{\tau_{bT}}{\tau_{b20^\circ C}} \right) \left(1 - \frac{l_0}{l} \right) \right] \tag{25}$$

where:

τ_{bT} = residual bond strength after exposure of temperature T (after cooling);
$\tau_{b20^\circ C}$ = residual bond strength at ambient temperature;
l_0 = span length for constant moment zone = $9.3 \times x_T$ (Au and Du, 2004).

In the RC beam due to fire exposure, bond degradation occurs and hence this partial bond between concrete and steel rebar may influence the flexural capacity of fire-damaged RC beams. Because of the partial bond between fire-affected concrete and steel rebar, the tensile strain cannot be fully transferred to reinforcement due to insufficient bond, then the tensile reinforcement can only provide the following flexural tensile force (F_T):

$$F_T = n \cdot \tau_{bT} \cdot \left(\Pi \cdot \Phi \cdot \frac{l}{2} \right) \leq \left(n \cdot \frac{\Pi}{4} \Phi^2 f_{yT} \right) \tag{26}$$

where:

ϕ = diameter of steel rebar;
l = total length of tensile rebar;
τ_{bT} = average residual bond strength after exposure to temperature T (after cooling):

$$=\frac{1}{2}(\cdot\ \tau_{b\,max,20^{\circ}C})k_{\tau bT}\ \text{to}\ \left(\frac{2}{3}\cdot\tau_{b\ max,\ 20^{\circ}C}\right)k_{\tau bT} \tag{27}$$

$k_{\tau bT}$ = residual bond strength reduction factor due to fire exposure;
$\tau_{b\ max,\ 20^{\circ}C}$ = maximum bond strength between concrete and steel rebar at ambient temperature $= 2.5\sqrt{f_c}$ (CEB, 1990).

The value of steel stain after fire can be calculated as:

$$\varepsilon_{sT} = \frac{F_T}{A_S E_{sT}} \tag{28}$$

and the value of strain in concrete at extreme fiber can be obtained by making use of modified stain compatibility relationship using equations (24)-(25) as:

$$\varepsilon_{cT} = g(\tau_{bT})\cdot\left(\frac{d - x_T}{x_T}\right)\cdot\varepsilon_{sT} \tag{29}$$

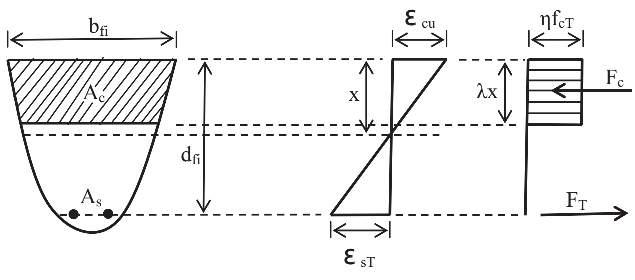
With reference to Figure 4, the depth of stress block is taken as λ*x*, where *x* is the depth of the neutral axis, and λ is given by:

$$\lambda = 0.8 - \frac{f_c - 50}{400} \leq 0.8 \tag{30}$$

And the concrete strength is taken as ηf_c, where η is given by:

$$\eta = 1.0 - \frac{f_c - 50}{200} \leq 1.0 \tag{31}$$

If $\varepsilon_{sT} \leq \varepsilon_{yT}$ and $\varepsilon_{cT} \leq \varepsilon_{cuT}$, then the fire-damaged beam may suffer anchorage failure.



Source: Euocode 2 (2004b)

Figure 4.
Rectangular stress distribution

The depth of the neutral axis (x) can be calculated by equating total compressive force provided by concrete to flexural tensile force as:

$$x = \frac{\tau_{bT} \cdot \left(\Pi \cdot \Phi \cdot \frac{1}{2}\right)}{\lambda \cdot \eta \cdot f_{cT} \cdot b_{fi}} \tag{32}$$

where, the effective cross-sectional size (b_{fi} and d_{fi}) can be obtained by making use of 500°C isotherm method or by slice method.

The residual moment capacity of fire-damaged singly reinforced concrete beam after cooling can be taken as:

$$M_u = F_T * \left(d_{fi} - \frac{\lambda x}{2}\right) \tag{33}$$

If $\varepsilon_{sT} \leq \varepsilon_{yT}$, then it is assumed that the steel yields before anchorage failure occurs as the bond strength may be adequate enough for the stress transfer between the steel and concrete.

Then, the ultimate tensile force of the reinforcing bar is:

$$F_T = n \cdot f_{yT} \cdot \left(\frac{\Pi\Phi^2}{4}\right) \tag{34}$$

where:

n =number of tensile steel rebars.

The depth of neutral axis (x) can be calculated by equating total compressive force provided by concrete to flexural tensile force as:

$$x = \frac{n \cdot f_{yT} \cdot \left(\frac{\Pi\Phi^2}{4}\right)}{\lambda \cdot \eta \cdot f_{CT} \cdot b_{ft}} \tag{35}$$

The residual moment capacity of fire-damaged singly reinforced concrete beam after cooling can be taken as:

$$M_u = F_T * \left(d_{fi} - \frac{\lambda_x}{2}\right) \tag{36}$$

If $\varepsilon_{cT} > \varepsilon_{cuT}$, then it is assumed as that crushing failure occurs.

Now, by putting $\varepsilon_{cT} > \varepsilon_{cuT}$, ε_{sT} is obtained from equations (24)-(25). Then, the tensile force provided by steel is:

$$F_T = n \cdot \varepsilon_{sT} \cdot E_{sT} \cdot \left(\frac{\Pi\Phi^2}{4}\right) \tag{37}$$

The depth of the neutral axis (x) can be calculated by equating total compressive force provided by concrete to flexural tensile force as:

$$x = \frac{n \cdot \varepsilon_{sT} \cdot E_{ST} \cdot \left(\frac{\Pi\Phi^2}{4}\right)}{\lambda \cdot \eta \cdot f_{CT} \cdot b_{fi}} \tag{38}$$

The residual moment capacity of fire-damaged singly reinforced concrete beam after cooling can be taken as:

$$M_u = F_T * \left(d_{fi} - \frac{\lambda x}{2}\right) \tag{39}$$

In case of a doubly reinforced beam, it is assumed that the compression reinforcement also yields when the section reaches its capacity. For prediction of moment capacity of such fire-damaged beam, it can be divided into an imaginary beam and a coupling reinforcement as in Figure 5.

The residual moment capacity (M_u) of doubly reinforced concrete beam after fire exposure can be calculated as:

$$M_u = M_{u1} + M_{u2} \tag{40}$$

where:

M_{u1} = residual moment capacity of an imaginary beam after fire exposure, calculated in a similar way as it is done for singly reinforced fire-damaged beam.

M_{u2} = residual moment capacity of a coupling beam after fire exposure.

$$= A_s f_{scyT}(z') \tag{41}$$

7. Test data

A total of 23 test results from six studies (Kodur *et al.*, 2010; Prasad *et al.*, 2010; Prasad *et al.*, 2012; Mahdi *et al.*, 2009; Kumar, 2003; Moetaz *et al.*, 1996) were collected for fire-damaged RC beams after cooling and then tested for their residual load/moment capacities determination experimentally (tested under four-point loading test condition, except in Prasad *et al.*'s (2010) study, where three-point loading test condition test was used). The details of reference beams are given in Table I.

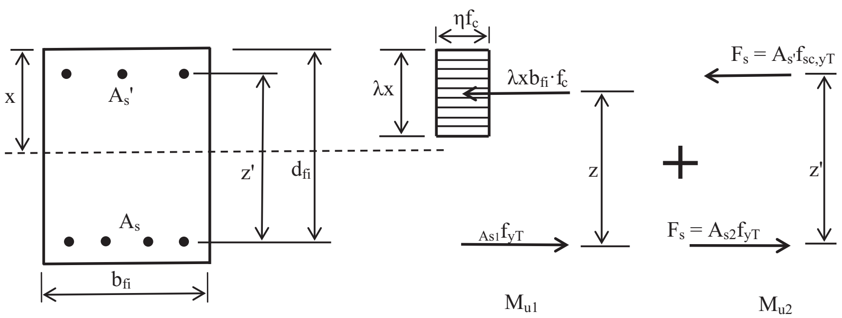


Figure 5. Stress distribution at ultimate limit state for a rectangular concrete cross section with compression reinforcement

Fire parameters T_f (°C)	t (hrs)	b (mm)	D (mm)	l (mm)	l_0 (mm)	c (mm)	f_c (MPa)	Tension reinforcement n-ϕ (mm)	Tension reinforcement f_y (MPa)	Compression reinforcement n-ϕ (mm)	Compression reinforcement f_y (MPa)
**Yuye *et al.* (2012)											
1050	2.5	250	400	4,000	1,330	30	28.5	3-25	457.5	2-14	472.5
*Kodur *et al.* (2010)											
1100	1.0	254	406	3,952	860	41	52.2	3-19	420	2-13	420
1100	1.0	254	406	3,952	860	41	93.3	3-19	420	2-13	420
1100	2.0	254	406	3,952	860	41	93.3	3-19	420	2-13	420
*Prasad *et al.* (2010)											
700	0.5	100	200	1,970	–	20	48	2-16	415	2-8	415
800	1.0	100	200	1,970	–	20	48	2-16	415	2-8	415
950	1.5	100	200	1,970	–	20	48	2-16	415	2-8	415
700	0.5	100	200	1,970	–	20	24	2-16	415	2-12	415
800	1.0	100	200	1,970	–	20	24	2-16	415	2-12	415
950	1.5	100	200	1,970	–	20	24	2-16	415	2-12	415
*Mahdi *et al.* (2009)											
400	1.0	100	100	900	300	20	42	2-8	540	2-6	540
400	2.0	100	100	900	300	20	42	2-8	540	2-6	540
700	1.0	100	100	900	300	20	42	2-8	540	2-6	540
700	2.0	100	100	900	300	20	42	2-8	540	2-6	540
400	1.0	100	100	900	300	20	89	2-8	540	2-6	540
400	2.0	100	100	900	300	20	89	2-8	540	2-6	540
700	1.0	100	100	900	300	20	89	2-8	540	2-6	540
700	2.0	100	100	900	300	20	89	2-8	540	2-6	540
**Kumar (2003)											
1006	1.0	200	300	3,660	1,330	25	17	4-12	415	2-10	415
945	1.5	200	300	3,660	1,330	25	17	4-12	415	2-10	415
*Moetaz *et al.* (1996)											
650	0.5	120	200	1,800	600	20	20	2-10	358.5	2-10	235
650	1.0	120	200	1,800	600	20	20	2-10	358.5	2-10	235
650	2.0	120	200	1,800	600	20	20	2-10	358.5	2-10	235

Notes: *Exposed to design fire; **Exposed to standard fire

Table I.
Reference beam details

8. Results and discussion

The residual moment capacities of an RC beam exposed to fire after cooling have been predicted by using 500°C isotherm method, modified 500°C isotherm method and slice method after making assumption that a perfect bond exists between steel rebar and surrounding concrete even after fire exposure, and these predicted residual moment capacities values are compared with the experimental results (Table II).

The slip approach with partial bond assumption between steel rebar and surrounding concrete in fire-damaged RC beams after cooling has also been used to predict the residual moment capacities of such beams by using 500°C isotherm method, and these predicted residual moment capacities values are also compared with the experimental results (Table III):

- From Tables II and III, it can be observed that the predicted residual moment capacities of beams exposed to fire for a relatively long period, the percentage deviation, calculated for the experimental results and predicted values by any of the methods, increases with the increase of temperature and also with the increase of

Table II. Comparison among predicted and experimental moment capacities with perfect bond assumption

Reference beams	Temp. (°C)	Time (hrs)	Experimental moment capacity (kNm)	500°C isotherm Predicted moment capacity (kNm)	Modified 500°C isotherm Predicted moment capacity (kNm)	Slice method Predicted moment capacity (kNm)
Yuye *et al.* (2012)	1,050	2	167.1	194.27	179.74	168.03
*Kodur *et al.* (2010)	1,100	1	153.6	118.89	116.69	113.13
	1,100	1	163.59	119.72	118.14	114.81
	1,100	2	142.31	113.16	112.01	105.74
Prasad *et al.* (2010)	700	0.5	26.98	26.06	25.03	25.14
	800	1	16.49	24.27	22.97	24.26
	950	1.5	17.10	21.13	19.24	21.25
	700	0.5	16.56	25.47	24.8	24.72
	800	1	14.43	23.96	23.2	23.85
	950	1.5	13.74	21.19	20.23	20.98
Mahdi *et al.* (2009)	400	1	3.98	3.33	3.31	3.28
	400	1.5	3.84	3.28	3.24	3.31
	700	1	2.64	3.23	3.21	3.13
	700	1.5	1.8	3.09	3.05	3.04
	400	1	5.03	3.81	3.37	3.31
	400	1.5	4.13	3.35	3.33	3.34
	700	1	3.33	3.28	3.26	3.17
	700	1.5	2.19	3.14	3.12	3.07
Kumar (2003)	945	1	46.5	43.96	42.4	40.32
	1,006	1.5	39.3	40.84	39.23	36.33
Moetaz *et al.* (1996)	650	0.5	14.81	8.84	8.77	8.63
	650	1	13.54	8.81	8.73	8.6
	650	2	10.5	8.33	8.23	8.23

Note: *Experimental moment capacity = Moment capacity due to applied load (or the residual load capacity) after cooling + moment due to applied load during heating

Table III. Comparison between predicted and experimental moment capacity by slip approach with partial bond assumption

Reference beams	Temp (°C)	Time (h)	f_c (mPa)	Experimental moment capacity (kNm)	Predicted moment capacity (kNm)	ΔM (%)
Yuye *et al.* (2012)	1,050	2	28.475	167.1	157.48	−5.75
**Kodur *et al.* (2010)	1,100	1	52.2	153.6	133.63	−12.99
	1,100	1	93	163.59	139.56	−14.68
	1,100	2	93	142.31	123.83	−12.98
Prasad *et al.* (2010)	700	0.5	48	26.98	25.21	−6.55
	800	1	48	16.49	20.20	22.54
	950	1.5	48	17.1	12.41	−27.39
	700	0.5	24	16.56	24.85	50.06
	800	1	24	14.43	17.53	21.50
	950	1.5	24	13.74	8.81	−35.85
Mahdi *et al.* (2009)	400	1	42	3.98	4.67	17.34
	400	1.5	42	3.84	4.59	19.53
	700	1	42	2.64	3.39	28.41
	700	1.5	42	1.8	2.85	58.33
	400	1	89	5.03	5.10	1.39
	400	1.5	89	4.13	5.01	21.31
	700	1	89	3.33	3.93	18.02
	700	1.5	89	2.19	3.45	57.53
Kumar (2003)	945	1	17	46.5	47.10	1.29
	1,006	1.5	17	39.3	39.90	1.53
Moetaz *et al.* (1996)	650	0.5	21.25	14.81	13.12	−11.41
	650	1	21.25	13.54	12.50	−7.68
	650	2	21.25	10.5	11.27	7.33

Note: **Experimental moment capacity = Moment capacity due to applied load (or the residual load capacity) after cooling + moment due to applied load during heating exposure period to the fire. This may possibly have happened because long-term fire might have damaged the beam structure badly. So, the proposed model should be applied conservatively in such cases.

- The percentage deviation, calculated for the experimental results and predicted values, is more obvious in case of (Mahdi *et al.*, 2009 and Prasad *et al.*, 2003). The possible reason for this may be that the cross-sectional size of beams (Table I) used are lesser, as they should have the width as per the minimum width requirement for the making use of 500°C isotherm method, modified 500°C isotherm method and also for use of proposed slip approach (Annex B: EN 1992-1-2) (Eurocode 2, 2004a) and also because of using arbitrary fire curve in place of using standard fire curve or similar parametric fire curve (Table B1: EN 1992-1-2) (Eurocode 2, 2004a).

The non-uniform prediction behavior, i.e. neither entirely under-prediction nor over-prediction as shown in Figure 6, may be because of several factors associated with the prediction procedure such as fire curve used in fire simulation, exposure plus steady-state heating time, grade of concrete, type of aggregates used, water–cement ratio, the complex phenomenon of spalling, etc., may not have been addressed properly:

- The predicted results for reference beams mentioned in Kumar's (2003) and Yuye *et al.*'s (2012) studies were found to have good convergence with the experimental results. The

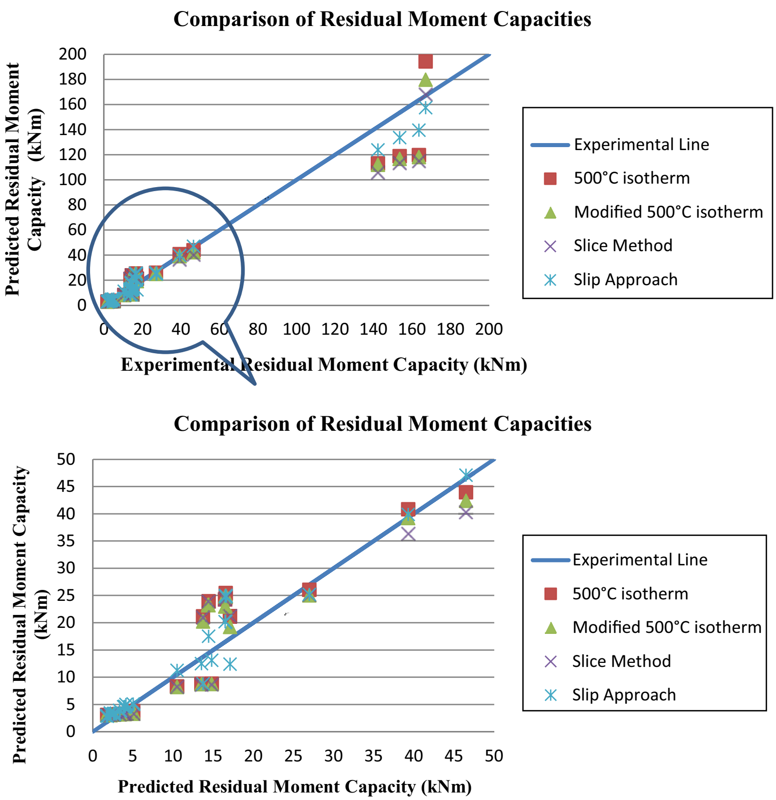


Figure 6.
Comparison of residual moment capacities of fire-damaged beams obtained from analytical models and experimental results of all the reference beams

probable reason may be that both have used a standard fire curve for the fire simulation in their respective experimental works and hence they satisfy the requirement of the proposed method.

- The predicted results by the proposed slip approach may be improved by making use of a more realistic strain compatibility relationship in case of partially bonded fire-damaged RC elements.

9. Concluding remarks

- The proposed slip approach for prediction of residual moment capacity of fire-damaged RC beam after cooling has better convergence than the prediction approaches with perfect bond assumption, as the proposed slip approach involves the assumption of existence of a partial bond between steel rebar and surrounding concrete which is more realistic one.
- The accuracy of the predicted results also significantly depends upon the nature of fire curve used.
- As the proposed slip approach for prediction of residual moment capacity of a fire-damaged beam in this study is applicable mostly in case of standard fire or similar parametric fire, hence, the proposed model should be applied conservatively in case of arbitrary fire curve.

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