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Fuzzy uncertainty and its applications in reinforced concrete structures

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Abstract

Purpose – The aim of this paper is mainly to handle the fuzzy uncertainties present in structures appropriately. In general, uncertainties of variables are classified as aleatory and epistemic. The different sources of uncertainties in reinforced concrete structures include the randomness, mathematical models, physical models, environmental factors and gross errors. The effects of imprecise data in reinforced concrete structures are studied here by using fuzzy concepts. The aim of this paper is mainly to handle the uncertainties of variables with unclear boundaries.

Design/methodology/approach – To achieve the intended objective, the reinforced concrete beam subjected to flexure and shear was designed as per Euro Code (EC2). Then, different design parameters such as corrosion parameters, material properties and empirical expressions of time-dependent material properties were identified through a thorough literature review.

Findings – The fuzziness of variables was identified, and their membership functions were generated by using the heuristic method and drawn by MATLAB R2018a software. In addition to the identification of fuzziness of variables, the study further extended to design optimization of reinforced concrete structure by using fuzzy relation and fuzzy composition.

Originality/value – In the design codes of the concrete structure, the concrete grades such as C16/20, C20/25, C25/30, C30/37 and so on are provided and being adopted for design in which the intermediate grades are not considered, but using fuzzy concepts the intermediate grades of concrete can be recognized by their respective degree of membership. In the design of reinforced concrete structure using fuzzy relation and composition methods, the optimum design is considered when the degree of membership tends to unity. In addition to design optimization, the level of structural performance evaluation can also be carried out by using fuzzy concepts.

Keywords Fuzzy set theory, Membership functions, Fuzzy set operations, Fuzzy relation, Fuzzy composition, Design optimization, Reinforced concrete

Paper type Research paper

1. Introduction

Uncertainties are usually classified in aleatory (random) and epistemic (fuzzy). Aleatory uncertainty arises from the inherent randomness in the physical properties and the system environment (Li *et al.*, 2016; Kiureghian and Ditlevsen, 2009; Pascal, 1975), whereas epistemic uncertainty originates from a lack of sufficient knowledge and imprecision of information about a system going to be studied. The type of uncertainty and the way of dealing with them have been addressed by many investigators (Marano and Quaranta, 2008; Du *et al.*, 2006; Brown *et al.*, 1983; Bulleit, 2008; Nikolaidis *et al.*, 2004) to solve problems in

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their respective areas of specialization. The aleatory uncertainty estimation problem is usually carried out by using probability theory, which requires a large number of samples, whereas epistemic uncertainty is usually modeled by possibility theory, which requires a small sample.

Civil engineering structures are complex and usually bulky in nature; hence, they will be almost impossible to test the prototype rather than checking specific criteria on uncertain software models and limited numerical methods that are based on human knowledge to solve a real problem. Usually, uncertainties of parameters in the concrete structure can be identified (Holicky, 2009) as inherent randomness of variables, statistical model uncertainties, physical model uncertainties, the vagueness of human knowledge and gross errors in design, execution and operation along with design life of the structure. In reinforced concrete structure, there exist numerous amounts of fuzzy uncertainties in material properties, section properties, applied loads and section capacities.

The human reasoning to counteract these complexities, uncertainties, imprecision and vagueness of data in professional judgment leads the main motivation to use fuzzy concepts. To handle this problem, Zadeh(1965) introduced a fuzzy set theory, which holds a continuum of the degree of membership to model the vagueness. Many researchers proved that (Fan *et al.*, 2019; Tang *et al.*, 2014; Bagheri *et al.*, 2017; Sarkar *et al.*, 2016; Yeh and Hsu, 1990) the fuzzy theory is an important tool and effectively well-designed for the problems in structural engineering to perform the reliability analysis and optimal design solution. Fuzzy concepts resemble human reasoning through providing an easy way of handling with real problems because of their simplicity and flexibility, easy to handle problems with imprecise and incomplete data, ability to deal with uncertainty and nonlinearity, cover a wider range of operating conditions and more readily customizable in natural language terms or linguistic terms. This paper tried to determine the fuzziness of reinforced concrete material properties, their membership functions and showed the design optimization and/or section performance by using the fuzzy relation and fuzzy composition.

2. Literature review

2.1 Fuzziness and fuzzy set theory

Fuzziness means an expression having an uncertain extensional denotation that has an ambiguous boundary (Zhang, 1998) and also arises from inconsistency or error. The function of the fuzziness is called a *measure of fuzziness*. Fuzziness can be applied in civil engineering discipline with linguistic variables, whose values are not numbers but words or sentences in a natural or artificial language as (Zimmermann, 2011) *very cold*, *cold*, *warm*, *hot* and *very hot* for temperature; *low*, *moderate* and *high* for corrosion rate; *under reinforced*, *balanced* and *over-reinforced* for reinforced concrete section; *no damage*, *slight damage*, *moderate damage*, *severe damage and destructive damage* for damage assessment of earthquake effect on structures with unclear boundaries.

Fuzzy set theories are suitable tools used for professional decision-making in structural engineering specialization such as risk assessment, reliability analysis, design optimization and performance evaluation of the structures. These decisions are expressed in linguistic terms (e.g. "the structure is '*slightly* damaged" or "the quality control is not adequate") with fuzziness which avoids the usual conventional set representation (Brown *et al.*, 1983). For this reason, *fuzziness* may be experienced to solve an ambiguous "how" questions, such as: *How severe damage is severe*? or *How under reinforced is under*?, in general, "*How many is many*?" questions. This kind of question aims for the referential meaning of an expression and has a clear boundary to answer the question (Zhang, 1998).

2.1.1 Fuzzy sets. The fuzzy sets concept was introduced by its pioneer (Zadeh, 1965) to represent variables with imprecise or ambiguous boundaries. Therefore, the fuzzy set theory is used to handle ambiguity, vagueness, imprecision and insufficient level of expert knowledge on real-life phenomena as a source of uncertainty.

In ordinary set theory (Mazeika *et al.*, 2007), the element that fulfills some defined conditions by a set is only considered as members of this set. In this case, the degree of membership is binary, i.e. either zero or one, which indicates that the element belongs to the set or it does not. Therefore, in ordinary set theory, there are well-defined boundaries to identify an object belongs to a set or does not.

On the other hand, the fuzzy set theory directly addresses the limitation of a crisp set by letting membership degree to which extent a variable belongs to a set. A fuzzy set is defined as that (Zadeh, 1965) is a class of objects with a continuum of grades of membership. In contrary to the crisp set, a fuzzy set is prescribed by vague or ambiguous properties; hence, its boundaries are ambiguously specified. The fuzzy set theory is an important tool (Brown *et al.*, 1983) that handles words and phrases of linguistic variables numerically by using the membership functions.

According to Holicky and Schneider's(2002) notion, the fuzzy set will be represented as a set of ordered pairs of elements; each presents the element together with its membership value. Usually, a fuzzy set is represented as "$\tilde{A}$" whereas a crisp set is represented as "A". A fuzzy set can be represented mathematically for both as based finite and infinite elements. The elements of the discrete fuzzy set $\tilde{A}$ can be represented with its membership function as:

$$\tilde{A} = \left\{ \left(x, \mu_{\tilde{A}}(x) \right) \right\} \text{ or } \tilde{A} = \sum_{i=1}^{n} \mu_{\tilde{A}}(x_i)/x_i \tag{1a}$$

For elements of the continuous fuzzy set as:

$$\tilde{A} = \int \mu_{\tilde{A}}(x_i)/x_i \tag{1b}$$

where $\mu_{\tilde{A}}$ is the membership function or grade of membership of x in $\tilde{A}$ that maps X to the membership space, and in the expressions, the symbol "Σ" or "$\int$" implies not addition or integration, respectively, but union.

2.1.2 Membership function. In fuzzy set theory, a fuzzy set $\tilde{A}$ in the universe of discourse X is characterized by a characteristic function $\mu_{\tilde{A}}(x)$ (Zadeh, 1965), which associates with each point in the universe of discourse X, a real number in the interval [0, 1], with the value of $\mu_{\tilde{A}}(x)$ at x representing the degree of membership of x in $\tilde{A}$ is called *membership function*. The universe of discourse X in concrete cases has to be chosen according to a real problem in a specific situation. The *membership function* indicates the transition of an object from not belonging to belonging is gradual, which helps us to handle impreciseness and vagueness of variables. Mathematically, the degree of membership can be expressed by:

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 < \mu_{\tilde{A}}(x) \leq 1 & \text{for } x \in A \\ 0 & \text{for x} \notin \text{A} \end{cases} \tag{2}$$

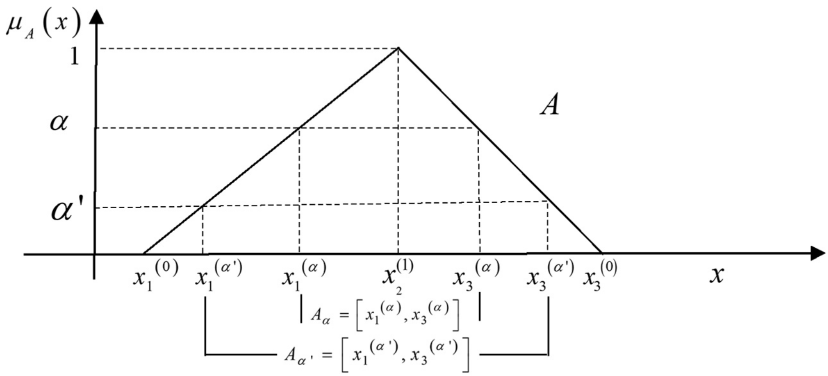
in which "0" means complete exclusion from the set $\tilde{A}$, $''0 < \mu_{\tilde{A}}(x) < 1''$ means partial inclusion form the set $\tilde{A}$ and "1" means absolute inclusion in the set $\tilde{A}$. Thus, the nearer the value of $\mu_{\tilde{A}}(x)$ to *unity*, the higher the degree of membership of x in $\tilde{A}$.

Membership functions are the crucial component of fuzzy set theory, i.e. fuzziness in a fuzzy set is determined by its membership function. Accordingly, the shapes of membership functions are a useful tool for a particular problem, as they affect a fuzzy inference system. It was introduced (Zadeh, 1965; Zadeh, 1978) and initially widely accepted, as membership functions are subjective and based context of the events, latter from measurement view (Bilgiç and Türkşen, 2000), it is the connection of both subjective and objective to make a sound decision. There are numerous types of membership functions such as triangular, trapezoidal, Gaussian, singleton, bell curves, sigmoidal functions. Nevertheless, the only condition a membership function must really satisfy is that it must vary between zero and one. To make the best choice, one needs a lot of experience with the given situation. The frequently applied membership function is the triangular membership functions, which are formed by using straight lines. These straight-line membership functions have the advantage of simplicity.

2.1.3 Fuzzy numbers. The fuzzy number is expressed as a fuzzy set defining a *fuzzy interval* in the real number $\Re$. As the boundary of a *fuzzy interval* is ambiguous, the interval is also a *fuzzy set*. Generally, a *fuzzy interval* is represented by two endpoints $x_1^{-(0)}$ and $x_3^{+(0)}$ and a peak point $x_2^{(1)}$, as shown in Figure 1. *Fuzzy numbers* are a special case of fuzzy sets that have to satisfy (Lee, 2004) all the conditions: convex fuzzy set; normalized fuzzy set; its membership function is piecewise continuous and it is defined in the real number. Mathematically, *fuzzy numbers are* expressed as:

$$X = \{(x, \mu_X(x)) : x \in \Re; \mu_X(x) \in [0, 1]\} \tag{3}$$

where X is the fuzzy number; $\mu_X(x)$ is the membership value of the element x to the fuzzy number X and $\Re$ is the set of real numbers.



Source: Lee (2004)

Figure 1.
α-cut fuzzy number
$(\alpha' < \alpha) \Rightarrow (A_{\alpha \subset A\alpha'})$

The condition of normalization in the fuzzy set implies that the maximum membership value is 1.

$$\exists x \in \Re, \qquad \mu_{\tilde{A}}(x) = 1$$

The convex condition of a fuzzy set is that the line by *α-cut* is continuous and *α-cut* interval satisfies the following relation.

$$A_\alpha = \left[x_1{}^{(\alpha)}, x_3{}^{(\alpha)}\right] \tag{4}$$

$$(\alpha' < \alpha) \Rightarrow \left(x_1{}^{(\alpha')} \leq x_1{}^{(\alpha)}, x_3{}^{(\alpha')} \geq x_3{}^{(\alpha)}\right)$$

The convex fuzzy set condition may also be written as (Lee, 2004) $(\alpha' < \alpha) \Rightarrow (A_\alpha \subset A_{\alpha'})$. If all the *α-cut* sets are convex, the fuzzy set with these *α-cut* sets is convex (Figure 1). In other words, if a relation is given as:

$$\mu_{\tilde{A}}(x) \geq \min\left[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right] \tag{5}$$

where $x = \lambda x_1 + (1-\lambda)x_2,\ x_1, x_2 \in \Re^n, \lambda \in (0,1)$holds the *fuzzy* set $\tilde{A}$ is *convex*. A *fuzzy* variable X with the membership function $\mu_X(x)$ is *strongly convex* if and only if the event $\{x|\ \mu_X(x) \geq \alpha\}$ is strongly convex $\forall\ \alpha\ \varepsilon$ (0.1).

2.1.4 Fuzzy set operations. The pioneer of the fuzzy set concept (Zadeh, 1965) also induced the combination of membership functions and several properties involving fuzzy sets which are noticeable extensions of the corresponding definitions for conventional sets. The membership function is clearly the crucial part of a fuzzy set. It is, therefore, operations with fuzzy sets are defined through their membership functions. The most widely used operations are called *standard fuzzy set operations* which are complements, intersections and union.

- *Complement*: The *complement* of a fuzzy set $\tilde{A}$ is denoted by$\tilde{A}'$ and is defined by:

$$\mu_{\tilde{A}'}(x) = 1 - \mu_{\tilde{A}}(x), x \in X \tag{6}$$

- *Union*: The *union* of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ is a fuzzy set $\tilde{C}$, written as $\tilde{C} = \tilde{A} \cup \tilde{B}$, whose membership function is related to those of $\tilde{A}$ and $\tilde{B}$ by:

$$\mu_{\tilde{C}}(x) = \mathrm{Max}\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right\}, x \in X \tag{7}$$

- *Intersection:* The *intersection* of two fuzzy sets $\tilde{A}$ and $\tilde{B}$ with respective membership functions $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ is a fuzzy set $\tilde{C}$, written as $\tilde{C} = \tilde{A} \cap \tilde{B}$, whose membership function is related to those of $\tilde{A}$ and $\tilde{B}$ by:

$$\mu_{\tilde{C}}(x) = \mathrm{Min}\left\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(x)\right\}, x \in X \tag{8}$$

2.2 Operation of fuzzy interval and α-cut interval

The α-cut interval of fuzzy number is crisp set, and the operation of a fuzzy number can be generalized from that of a crisp interval. Suppose $A = [x_1, x_3], B = [y_1, y_3], \forall x_1, x_3, y_1, y_3 \in \Re$ that A and B as numbers articulated as fuzzy interval, the main operations of interval are (Lee, 2004) addition (+) and subtraction (−) in which the shape of membership function will not be changed; multiplication (•) and division (/), in which the shape of membership function will be changed and inverse interval, $[x_1,x_3]^{-1} = [1/x_1 \wedge 1/x_3, 1/x_1 \vee 1/x_3,]$ excluding the case $x_1 = 0$ or $x_3 = 0$. In addition to these operations, *Minimum*: $[x_1,x_3](\wedge)\,[y_1, y_3] = [x_1 \wedge y_1, x_3 \wedge y_3]$ and *Maximum*: $[x_1, x_3](\vee)\,[y_1, y_3] = [x_1 \vee y_1, x_3 \vee y_3]$ operations can be used in the fuzzy interval.

The α-cut set is the crisp set of the elements whose degree of membership is greater than or equal to α, (Lee, 2004; Zimmermann, 2011) i.e. $X_\alpha = \{x, \mu_X(x) \geq \alpha : x \in \Re; \alpha \in [0,1]\}$ and $X_\alpha = \{\mathrm{x}, \mu_X(x) > \alpha\}$ is *strong α-cut set* in which X_α is the crisp set at the α-level set and α is the credibility level. Let us consider the fuzzy set of the concrete strength in N/mm^2:

$$\tilde{F}_{ck} = \{0/20 + 0.2/21 + 0.4/22 + 0.6/23 + 0.8/24 + 1/25 + 0.8/26 + 0.6/27 + 0.4/28 + 0.2/29 + 0/30\}$$

This fuzzy set of concrete strength can be set into several *α-cut* and *strong α-cut sets*, all of which are crisp for the arbitrary values of α = 1, 0.8, 0.6, 0.4 and 0.

The *α-cut sets* are as follows:

$F_{ck1} = \{25\}$ $F_{ck0.8} = \{24\ 25\ 26\}$ $F_{ck0.6} = \{23\ 24\ 25\ 26\ 27\}$
$F_{ck0.4} = \{22\ 23\ 24\ 25\ 26\ 27\ 28\}$ $F_{ck0.2} = \{21\ 22\ 23\ 24\ 25\ 26\ 27\ 28\ 29\}$

Strong α-cut sets are as follows:

$F_{ck0.8} = \{25\}$ $F_{ck0.6} = \{24\ 25\ 26\}$ $F_{ck0.4} = \{23\ 24\ 25\ 26\ 27\}$ $F_{ck0.2} = \{22\ 23\ 24\ 25\ 26\ 27\ 28\}$

The value of α is the threshold for two adjacent data to be thought of as belonging to the same class in which small α will have the large fuzzy interval and the large α will have a smaller fuzzy interval.

For a *fuzzy number* denoted by X and a real number denoted by r_0, Figure 2(a) and (b) show their possible order relations using *α-cut*. The mathematical notations, $\mu_X\,(r)$ represent the membership function of X; m is the mean of X with $\mu_X\,(m) = 1$; the vertical axis denotes the degree of membership; α_0, α_1 and α_2 are three degrees of membership with the condition $\alpha_1 < \alpha_0 < \alpha_2$, respectively; α_0 is the membership degree of a real number r_0, i.e. $\alpha_0 = \mu_X\,(r_0)$, X_α represents the α-cut of X satisfying the condition $X_\alpha = \{r \mid \mu_X\,(r) \geq \alpha\}$ (Tang *et al.*, 2014). For each $\alpha_i\ \varepsilon(0.1]$, $X_\alpha = \{r \mid \mu_X\,(r) \geq \alpha i\}$ is an interval defined as $[x^{-\alpha_i}, x^{+\alpha_i}]$, as shown in Figure 2(a) and (b).

We can obtain three important points from Figure 2(a): the equality $x^{-\alpha_0} = r_0$supports; $x^{-\alpha}$ is larger than r_0 when $\alpha > \alpha_0$, such as α_2 where $\alpha_2 > \alpha_0$ and $x^{-\alpha_2} > r_0$ hold; and $x^{-\alpha}$ is less than r_0 when $\alpha < \alpha_0$, such as α_1 where $\alpha_1 < \alpha_1$ and $x^{-\alpha_1} < r_0$ hold. Thus, the critical degree of membership α_0 can be used to define the possibility that (Tang *et al.*, 2014) X is less than r_0, i.e. *POSS* $\{X \leq r_0\} = \mu_0$. To illustrate α-cut operations, assume the α-cut interval A_α and B_α of fuzzy number $\tilde{A}$ and $\tilde{B}$, respectively, is given as:

$$A_\alpha = \left[x_1{}^{(\alpha)}, x_3{}^{(\alpha)}\right], \forall \alpha \in [0,1], x_1{}^{(\alpha)}, x_3{}^{(\alpha)} \in \Re \tag{9}$$

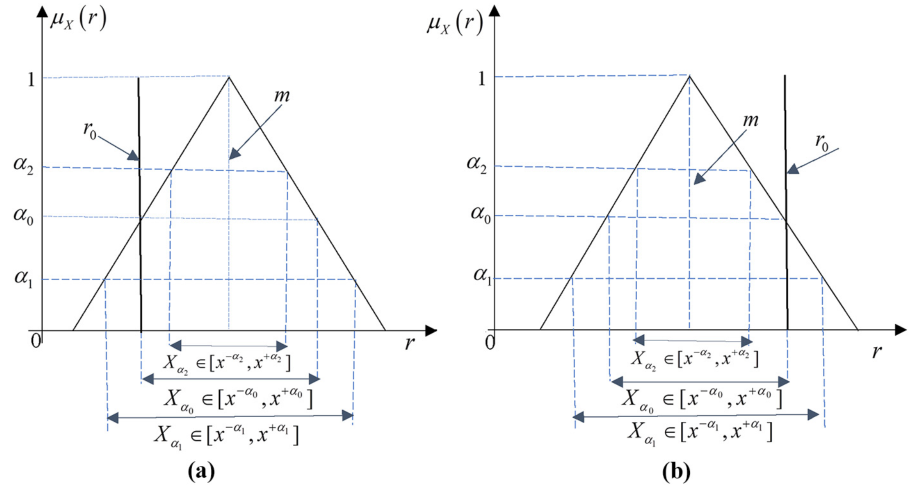


Figure 2. Order relation between X and r_0

$$B_\alpha = \left[y_1^{(\alpha)}, y_3^{(\alpha)}\right], \forall\alpha \in [0,1], y_1^{(\alpha)}, y_3^{(\alpha)} \in \Re \tag{10}$$

Then addition and subtraction operations of the α-cut interval between A_α and B_α can be expressed, as shown in equations (11) and (12) (Lee, 2004); these operations are also applicable to multiplication and division.

$$\left[x_1^{(\alpha)}, x_3^{(\alpha)}\right](+)\left[y_1^{(\alpha)}, y_3^{(\alpha)}\right] = \left[x_1^{(\alpha)} + y_1^{(\alpha)}, x_3^{(\alpha)} + y_3^{(\alpha)}\right] \tag{11}$$

$$\left[x_1^{(\alpha)}, x_3^{(\alpha)}\right](-)\left[y_1^{(\alpha)}, y_3^{(\alpha)}\right] = \left[x_1^{(\alpha)} - y_1^{(\alpha)}, x_3^{(\alpha)} - y_3^{(\alpha)}\right] \tag{12}$$

3. Objectives

The present study pictured to develop a relationship between input parameters, i.e. corrosion rate and an output parameter, i.e. compressive strength of concrete, and area of reinforcing steel, using triangular membership function in fuzzy set theory. The objective was to identify the membership function of the compressive strength of the concrete and effective area of reinforcing steel because of the corrosion of embedded reinforcement bar. It has further extended to the optimum design or performance evaluation of a reinforced concrete section using fuzzy membership functions of concrete strength and area of steel reinforcement that were developed in this paper.

4. Materials and methodology

Nowadays in urban and suburban areas, reinforced concrete structures are very common. However, the desired strength of concrete can be achieved during material selection, mixing, transportation, placement and curing because of an aggressive environment the corrosion of embedded reinforcement bar reduces the strength of concrete and reinforcing steel diameter significantly.

To illustrate the effect of corrosion, suppose a simply supported rectangular reinforced concrete beam of effective span 6 m, which has direct contact with soil which has the corrosion current density of 0.75 μA/cm^2, is subjected to a design bending moment of 206,71 kNm at the mid-span and shear force of 137. 74 kN at the center of support. Design is carried out based on EC2 (EN 1992-1-1, 2004) by using materials of concrete grade C25/30 and steel grade 460 N/mm^2 of the longitudinal bar, 250 N/mm^2 for transverse reinforcement (Figure 3).

5. Result and discussions

5.1 Generation of membership function of fuzzy variables

The generation of membership functions for imprecise data (Medasani *et al.*, 1998) is a basic stage in applications of the fuzzy concepts. The membership function of the fuzzy set can be generated by using different methods such as heuristics, the probability to possibility transformations, histograms, nearest neighbor techniques, feed-forward neural networks, clustering and mixture decomposition methods. However, these methods can be used to generate membership functions; there are no guidelines or rules to choose the appropriate membership generation technique because of lack of a consensus on the meaning on the membership function. The lack of a consensus of membership arises from that, fuzziness is subjective and decisions made by individuals.

In this paper, heuristic methods, which use predefined shapes for membership functions, are used to identify the membership of both input variables and output performance. Frequently used shapes of heuristic membership functions are *piecewise linear functions* and *piecewise monotonic functions*. In *piecewise linear functions*, the membership functions may be chosen to be linearly increasing, linearly decreasing or a combination of these and flat regions, i.e. triangular membership functions and trapezoidal membership functions. In the case of *piecewise monotonic functions*, membership functions have a (piecewise) smooth transition between non-membership and full-membership regions (Medasani *et al.*, 1998). The linear and piecewise linear membership functions give a reasonably smooth transition, easily handled by fuzzy operators and easily implemented.

The membership functions which have been generated from heuristic function have the following common features (Medasani *et al.*, 1998; Dombi, 1990):

- all membership functions are continuous;
- all membership functions map an interval $\mu[a,b] \rightarrow [0, 1]$; and
- membership functions are either monotonically increasing or monotonically decreasing or both increasing and decreasing.

The triangular membership function is defined by its lower limit *a*, its upper limit *b* and the modal value, *m* (if symmetrical), whereas for asymmetrical triangular membership, *m* is the

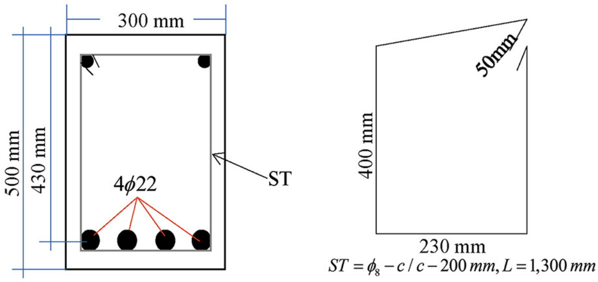


Figure 3.
Detail of beam section

mean value at which a membership degree is a *unity*, is also applicable, so that $a < m < b$. Mathematically, the triangular membership function is given by:

$$\mu(x) = \begin{cases} 0 & \text{if } x \leq a \\ (x-a)/(m-a) & \text{if } a \leq x \leq m \\ (b-x)/(b-m) & \text{if } m \leq x \leq b \\ 0 & \text{if } x \geq b \end{cases} \tag{13}$$

5.1.1 Corrosion rate. Suppose I_{corr} stands for corrosion current density. Let us classify the rate of corrosion current density as *low*, *moderate* and *high* of the linguistic variable. Val *et al.* (1998) identified the *low* rate, *moderate* rate and *high* rate of corrosion current density as subset of I_{corr} with the interval of i_{corr} = 0.1 – 0.5 μA/cm^2 for *low* corrosion rate; i_{corr} = 0.5 – 1 μA/cm^2 for *moderate* corrosion rate; and $i_{corr} > 1$ μA/cm^2 for *high corrosion* rate, in which i_{corr} = 1 μA/cm^2 is equal to i_{corr} = 11.6 μm/year.

The classical approach, probability, one way to define the classical set is a *low* rate. Suppose *low* rate, corrosion current density membership of *low* rate set which belongs to the universal set i_{corr} such that the member of corrosion current density is between 0.1 and 0.5 μA/cm^2. Similarly, the member of corrosion current density belongs to *moderate* rate, if it is between 0.5 and 1 μA/cm^2. Moreover, the member corrosion current density belongs to a *high* rate set when the corrosion current density is greater than 1 μA/cm^2. In the classical approach, it is obvious that 0.49 μA/cm^2 is a *low* rate according to the definition, while 0.51 μA/cm^2 is a moderate rate implying the classical sets have rigid boundaries and because of this, the expression of data becomes very difficult.

In a fuzzy set, it is very easy to represent them according to their respective membership degree.

As shown in Figure 4, if the corrosion current density is around 0.25 μA/cm^2, it is a *low* rate; the corrosion current density is around 0.75 μA/cm^2, it is a *moderate* rate and when the corrosion current density is around 1.25 μA/cm^2, it is *high* rate. In this sense, the fuzzy sets have no rigid boundary. Let us consider here, 0.5 μA/cm^2 can be simultaneously *low* rate as well as *moderate* rate, with a fuzzy membership grade of 0.5 (crossover point). When 0.625 μA/cm^2 is considered, it is likely in the category of *moderate* rate with a membership function of 0.75, whereas the 0.35 μA/cm^2 is a with a membership degree of 0.8 in *low* rate and 0.2 in *moderate* rate. In this study, the upper bound of corrosion current density was taken as 2 μA/

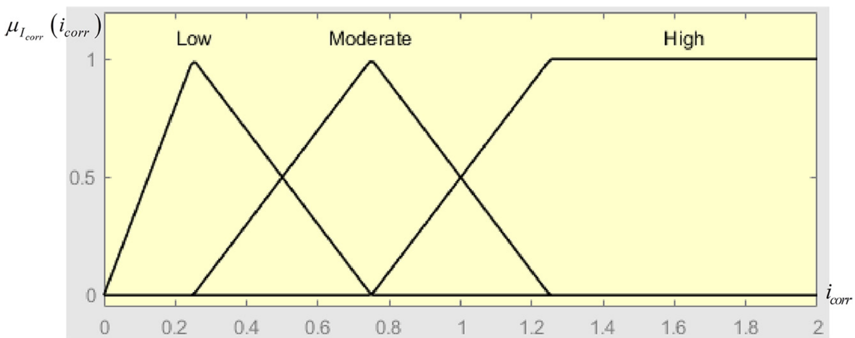


Figure 4. Fuzzy membership function of corrosion current density

cm^2 for illustration but its value may be more or less than 2 μA/cm^2 in a real problem. This is how the imprecise data can be categorized in a clear way by using fuzzy sets.

5.1.2 Effect of corrosion on the area of reinforcing steel and compressive strength of concrete. The effect of corrosion on the reinforcement steel has been investigated by many researchers (Stanish, 1997; Xia *et al.*, 2013; Zandi Hanjari *et al.*, 2008; Loreto *et al.*, 2011; Adukpo *et al.*, 2013; Zhou *et al.*, 2014) and found that: reduce load carrying capacity; loss of diameter or effective cross-sectional area; significantly reduce bond strength; increase crack width, and strongly reduced elongation of reinforcement steel.

Before corrosion takes place, the structure could mainly be subjected to the applied load. To prevent the effect of aggressive environment, design codes (EN 1992-1-1, 2004; IS456, 2000) provided respective concrete cover based on the exposure condition of the structural element. The concrete cover may delay the corrosion initiation time but does not fully control the corrosion. To consider the effect of corrosion on the serviceability of the structure, the corrosion initiation time is a very important factor. The corrosion initiation time depends on the concrete cover, chloride diffusion coefficient, chloride concentration percentage to the weight of concrete, concrete strength and the expression to determine the corrosion initiation time was derived by Thoft-Christensen *et al.*(1996) by using Fick's law of diffusion. The corrosion initiation time, T_i, can be obtained from the determining parameters C, C_0 and D_c give in expression (Thoft-Christensen *et al.*, 1996) as:

$$T_i = \frac{C^2}{4D_c}\left[erf^{-1}\left(\frac{C_0 - C_{cr}}{C_0}\right)\right]^{-2} \tag{14}$$

where C (cm) is the concrete cover thickness, D_c (cm^2/year) is the chloride diffusion coefficient, erf^{-1} is the inverse of error function computed by MATLAB, C_0 (Wt.% *of concrete*) is the equilibrium chloride concentration at the concrete surface, and C_{cr} (Wt.% *of concrete*) is the critical chloride concentration.

The time-variant resistance of the concrete section is then determined by considering the deterioration of the reinforcing steel diameter with reference to the corrosion initiation time. The reduction of the steel diameter is determined from the expression (Val *et al.*, 1998):

$$D_i(t) = \begin{cases} D_i & for\ t \leq T_i \\ D_i - r_{corr}(t - T_i) & for\ T_i \leq t \leq T_i + \dfrac{D_i}{r_{corr}} \\ 0 & for\ t \geq T_i + \dfrac{D_i}{r_{corr}} \end{cases} \tag{15}$$

where:

$D_i(t)$ = is the i^{th} diameter of the reinforcing bars at a time, t;
D_i = is the initial diameter of the i^{th} bar at the time;
n = is the number of the bars; and
r_{corr} = is the rate of corrosion in μm/year.

Similarly, the time-dependent compressive strength of concrete is given by expression (Kliukas *et al.*, 2015) as:

$$f_{cc}(t) = \alpha_{cc}k_2(t)f_{ck} \tag{16a}$$

In which:

$$\alpha_{cc}(t) = 1 - 0.1N_G/N_E \text{or } \alpha_{cc}(t) = 1 - 0.1M_G/M_E \tag{16b}$$

$$k_2(t) = 0.85 - 1.7\rho(t) \text{ and } \rho(t) = \frac{A_s(t)}{A_c} \tag{16c}$$

where:

N_G = is the permanent force;

N_E = is the transient force;

M_G = is the bending moment caused by permanent force;

M_E = is bending moment caused by permanent and transient loads $\rho(t)$ is time-dependent reinforcement ratio.

The baseline values from the reference, (Enright and Frangopol, 1998) taken as chloride diffusion coefficient D_c = 1.29 cm^2/year, Surface chloride concentration C_0 = 0.10 Wt.% of concrete, and critical chloride concentration C_{cr} = 0.04 Wt.% of concrete, the concrete cover of 5.8 cm is used for parametric studies. In this study of time-dependent safety analysis of simply supported reinforced concrete beam, using equation (14) the corrosion initiation time is 18.41 year. From detailed computation, the input variables and output variables are represented as shown in Table I.

The time-variant concrete strength and effective area of reinforcement steel because of moderate corrosion rate were computed, and the result at the design life is shown in Table I. As shown in Figure 4, the corrosion rate is a fuzzy variable which is considered as an input variable that deteriorates the strength of concrete, the diameter of reinforcement steel and however not significant also reduces the yield strength of reinforcement steel. The fuzziness of the corrosion rate is also propagated to the concrete and reinforcement steel that is also being fuzzy along with the design life of the concrete structure. Prior to corrosion, the concrete strength f_{ck} was 25 N/mm^2, and at the end of the design life, its strength degraded to 20.167 N/mm^2. Similarly, the area of reinforcement steel was initially 1520.531mm^2, and at the end of the design life, its area is 766.880mm^2 as shown in Table I. In design codes of concrete structure, the concrete grades are such as C20/25, C25/30 and C30/37, in which the intermediate grades are not recognized. However, the desired grade of concrete is achieved during construction, due to the aggressive environment the strength of concrete my decrease along with the design life of the structure and these intermediate strengths can be recognized by fuzzy set theory with their respective membership degrees.

The fuzziness of the variable is expressed by its degree of membership. To generate the membership function of the variable, it ranges i.e. the lower limit and upper limit have to be

Variables	Parameter	Abbreviation	Database range Minimum	Database range Maximum
Input	Corrosion current density (μA/cm^2)	i_{corr}	0.75	
	Chloride diffusion coefficient (cm2/year)	D_c	1.29	
	Surface chloride concentration (%)	C_0	0.10	
	Critical chloride concentration (%)	C_{cr}	0.04	
	Concrete cover (cm)	C	5.8	
	Time (year)	t	0	50
Output	Compressive strength of concrete (N/mm^2)	$f_{ck}(t)$	20.167	25
	Effective area of steel (mm^2)	$A_{st}(t)$	766.880	1520.531

Table I. Input and output variables

identified. Also, the range of variables has to be an appropriate limit to handle the variation of parameters easily and precisely. Therefore, the range of the concrete strength is limited to the lower and upper bound of the concrete strength of f_{ck} = 20 N/mm^2 and 30 N/mm^2 from EC2 (Table III). Similarly, the area of reinforcement steel in a singly reinforced concrete beam is limited to $A_{st,min} \leq A_{st,prov} \leq 0.02\,A_c$ as provided in EC2, in which $A_{st,min}$ = 0.25 $f_{ctm}bd/f_{yk}$ = 189.57 mm^2; $A_{st,prov}$ = 1520.53mm^2 and 0.02 A_c = 3000mm^2. However, the maximum limit of the steel area in a singly reinforced concrete beam is 0.02 A_c ;it should not be more than $A_{st,bal} = M_{bal}/(Z_{bal}\,f_{yk})$ =1935.04mm^2 in the under reinforced section. Therefore, the range of the reinforcement steel area is limited from $A_{st,min}$ -to- $A_{st,bal}$. Again, to generate the membership function the mean or modal value of the parameter, in which its degree of membership is maximum, which is unity in normalized case, has to be identified. To have that, the value obtained through the deterministic design procedure (Lu *et al.*, 1994) can be considered as the mean value of the study parameter. On this basis, the mean value of concrete strength is 25 N/mm^2 and that of reinforcement steel area is 1378.56mm^2.

5.1.3 Application of fuzzy set theory in reinforced concrete design. In structural engineering, the design parameters are not certain due to different factors. Some uncertainties can be estimated by preparing sufficient sample data, but human knowledge is limited to define and handle all uncertainties. The design optimization of the reinforced concrete structure possesses different constraints and bound of design variables (objectives). The lower and upper bound of the design variable implies the fuzzy interval of the variables. Within the fuzzy interval, the specific value of the variable has is the corresponding membership function. In reinforced concrete structure, the presence of fuzzy uncertainty of input variables such as material properties, cross-sectional properties, loading uncertainties and model uncertainties (Fan *et al.*, 2019) also propagates to the output performance of the structure by the function relationship between the input variables and output performance. In this paper, the optimum design of reinforced concrete structure based on the membership degree of input variables (concrete strength and reinforcement steel area) and the performance of the structure as output variable (flexure) was carried out by using fuzzy relation and fuzzy composition.

To illustrate the problem, let us consider the same problem used in Section 4 and take some elements of concrete strength and reinforcement steel with their respective degree of membership from Figure 5, and the flexural performance of the RC beam section. Let us take fuzzy sets of steel area as $\tilde{A}_{st} = \sum \mu_{\tilde{A}_{st}}(A_{st,i})/A_{st,i}$, the compressive strength of concrete

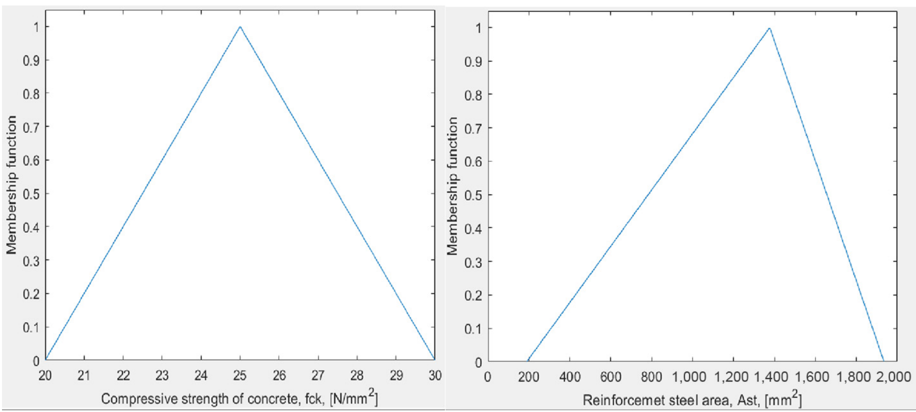


Figure 5.
Membership function of time-variant compressive strength of concrete and area of reinforcing steel due to corrosion

as $\tilde{F}_{ck} = \sum \mu_{\tilde{F}_{ck}}(f_{ck,j})/f_{ck,j}$ and flexural resistance of the beam section as $\tilde{M}_R = \sum \mu_{\tilde{M}_R}(M_{R,k})/M_{R,k}$. The fuzzy relation of steel area and the flexural resistance of the section is given by:

$$\tilde{R} = \tilde{A}_{st} \mathrm{x} \tilde{M}_R = \sum_{i}^{m} \sum_{k}^{p} \mu_R(A_{st,i}, M_{R,k})/(A_{st,i}, M_{R,k}) \tag{17}$$

in which $\mu_{\tilde{R}}(A_{st,i}, M_{R,k}) = \mu_{\tilde{A}_{st}}(A_{st,i}) \wedge \mu_{\tilde{M}_R}(M_{R,k}) = \min\left[\mu_{\tilde{A}_{st}}(A_{st,i}), \mu_{\tilde{M}_R}(M_{R,k})\right]$. Similarly, the fuzzy relation of concrete strength and the flexural resistance of the section is given by:

$$\tilde{S} = \tilde{F}_{ck} \mathrm{x} \tilde{M}_R = \sum_{i}^{m} \sum_{k}^{p} \mu_{\tilde{R}}(f_{ck,j}, M_{R,k})/(f_{ck,j}, M_{R,k}) = \min\left[\mu_{\tilde{F}_{ck}}(f_{ck,j}), \mu_{\tilde{M}_R}(M_{R,k})\right] \tag{18}$$

As if $\tilde{R}$ and $\tilde{S}$ are two fuzzy relations the fuzzy composition, which is used to obtain optimum design solution and to evaluate the performance of the structure (Brown *et al.*, 1983), is given by:

$$\tilde{R} \circ \tilde{S} = \left\{\max - \min\left[\mu_{\tilde{R}}(A_{st,i}, M_{R,k}), \mu_{\tilde{S}}(f_{ck,j}, M_{R,k})\right]\right\} \tag{19}$$

Let us consider the input variables of concrete strength and area of reinforcement with α-cut of 0.6 in which the combination of the lower and upper bound of variables to be considered. From the triangular membership function of concrete strength and area of reinforcement from Figure 5, let us consider the α-cut of 0.6 for these input variables. The fuzzy sets of concrete strength, area of reinforcement and the flexural capacity of the section which is computed from the section for corresponding strength of concrete and area of steel, respectively, as follows:

$$\tilde{F}_{ck} = \{0.6/23 \;\; 0.8/24 \;\; 1/25 \;\; 0.8/26 \;\; 0.6/27\}$$

$$\tilde{A}_{st} = \{0.6/902.96 \;\; 0.934/1300 \;\; 1/1378.56 \;\; 0.934/1415.29 \;\; 0.6/1601.15\}$$

$$\tilde{M}_R = \{0.62/141.13 \;\; 0.94/195.43 \;\; 1/206.71 \;\; 0.94/210.03 \;\; 0.52/237.42\}$$

All the possible combinations of input variables (concrete strength in N/mm^2 and area of reinforcement in mm^2) with their respective membership function and the flexural capacity in kNm of the reinforced concrete beam section are represented in Table II.

The triangular membership function of the flexural capacity was developed for the lower bound of 31.89 kNm, which from f_{ck} = 20 N/mm^2 and $A_{st,min}$ = 189.57mm^2, a mean value of 206.71 kNm and upper bound of $M_{\lim}$ = 273.01 kNm from f_{ck} = 25 N/mm^2 and $A_{st,bal}$ = 1935.04 mm^2.

The fuzzy relation of the area of reinforcement and flexural capacity of the section is obtained by the cross product of the column vector of the area of reinforcement and row vector flexural capacity by using equation (17). Similarly, the fuzzy relation of concrete

Table II. Membership function, the fuzzy interval of input variables and the section capacity

Material	Membership function	Interval	Flexural capacity interval of section	Membership function of flexural capacity
Concrete	0.6	[23, 27]	[141.13, 230.81]	[0.62, 0.62]
Steel	0.6	[902.96, 1601.15]	[143.23, 237.42]	[0.64, 0.52]
Concrete	0.6	[23, 27]	[194.21, 208.58]	[0.93, 0.96]
Steel	0.934	[1300,1415.15]	[198.56, 213.74]	[0.95, 0.89]
Concrete	0.6	[23, 27]	[204.06, 208.96]	[0.98, 0.95]
Steel	1	[1378.56, 1378.56]		
Concrete	0.8	[24, 26]	[141.72, 232.67]	[0.63, 0.60]
Steel	0.6	[902.96, 1601.15]	[142.77, 235.97]	[0.63, 0.54]
Concrete	0.8	[24, 26]	[195.43, 210.03]	[0.94, 0.94]
Steel	0.934	[1300,1415.15]	[197.60, 212.60]	[0.95, 0.90]
Concrete	0.8	[24, 27]	[205.44, 208.96]	[0.99, 0.97]
Steel	1	[1378.56, 1378.56]		
Concrete	1	[25, 25]	[206.71, 206.71]	[1, 1]
Steel	1	[1378.56, 1378.56]		

Note: In concrete steel combination, the interval of section capacity and membership function the first row and the second row of each combination rather than a membership grade is unity which was determined by considering the lower bound and the upper bound of concrete strength, respectively

strength and flexural capacity of the section is obtained by the cross product of a column vector of concrete strength and row vector flexural capacity by using equation (18). Then finally, from equation (19), the design optimization can be decided from the fuzzy composition of two relations. Using a similar procedure of fuzzy concepts, the level of time-dependent performance of the structure can also be evaluated:

$$\tilde{R} = \tilde{A}'_{st} x \tilde{M}_R = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.934 & 0.934 & 0.934 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.934 & 0.934 & 0.934 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix};$$

$$\tilde{S} = \tilde{F}'_{ck} x \tilde{M}_R = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix}$$

$$\tilde{D} = \tilde{R} \circ \tilde{S} = \begin{bmatrix} 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.62 & 0.94 & 1.00 & 0.94 & 0.52 \\ 0.62 & 0.80 & 0.80 & 0.80 & 0.52 \\ 0.60 & 0.60 & 0.60 & 0.60 & 0.52 \end{bmatrix} \rightarrow \text{is fuzzy composition}$$

As shown from the membership function of input variables and fuzzy composition matrix are closely related. When the degree of membership approaches to *unity* the design is good and the degree of membership approaches to zero (0) design becomes poor. For a single α-cut, there is one lower bound and one upper bound of both input variables and output variables except for $\alpha = 1$, which has a single value. If α-cut approaches to zero (0), the lower bound of the input variable is insignificant and the performance of structure becomes unsafe, for example, 0.6/23 of the concrete strength and 0.6/902.96 of area of the steel has the capacity of the section is 141.13 kNm which is very small compared with demand of the section i.e. 206.71 kNm. Whereas, if the upper bound of large then no doubt the performance of the section will be safe but it leads to over-strength deign and the economy would be under question. For example, the 0.6/27 concrete strength and 0.6/1601.15 area of steel of input variables the capacity of the section becomes 237.42 kNm, whose membership is 0.52, which is significantly larger than the demand of the section; consequently, the section becomes uneconomical. Rather than the membership degree of the core value of the structural performance, the other membership degrees have both lower and upper bounds of the section capacity. However, the degree of membership of the lower bound approaches to unity, the capacity is less than the demand of the section but the structure will not collapse because the reinforced concrete structure fails after attaining its possible plastic mechanisms.

6. Conclusion

The fuzzy uncertainty in which partial belonging of the parameter has been treated to handle imprecise information and vagueness of expert's knowledge. From the detailed analysis the following conclusions were drawn:

- In the case of corrosion rate, 0.5 μA/cm^2 corrosion current density can be either *low* rate or *moderate* rate in conventional set theory because it has no clear boundary for the linguistic terms. In fuzzy set theory, it can be handled wisely recognizing its membership grade as 0.5 for both *low* and *moderate* rates and the other intermediate rates can be recognized by their respective membership grades.
- The concrete strength was deteriorated from f_{ck} = 25 N/mm^2 to about 20 N/mm^2 in its design life which lost its degree of membership to almost zero (0), but this zero degree of membership does not mean to that the concrete has no strength instead its strength totally dropped to the next lower grade which has been provided in EC2.
- In optimum design, the combination of concrete strength with membership degree 0.6 and steel area with membership degree of unity (1) both the lower and upper bounds of flexural capacity are close to the demand of the section. This implies and strengthens the principle that the capacity of a singly reinforced section is governed by steel and wise consideration of fuzzy interval for a reasonable solution. In general, it can be concluded as the fuzzy set theory is applicable in design optimization or performance evaluation of reinforced concrete structure in which both safety and economy of the structure can be satisfied when the membership function of input variables approaches to *unity*.

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